# Applications of Monte Carlo Particle Filters in Signal Processing

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## 1 Introduction

Many problems in signal processing involve systems that are time varying in nature and are often described by signals whose models are nonlinear and include noise that is not necessarily additive and/or Gaussian. A standard approach for tracking the dynamics of signals in such problems has been the extended Kalman filter. In recent years, however, there has been a gradual shift of interest to a different methodology, known as Monte Carlo particle filters. Here we consider several applications of particle filters in signal processing including positioning, target tracking, navigation, blind deconvolution, and recursive estimation with discounted measurements.

A general description of the various problems is given by the following state-space model:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{f}_t(\mathbf{x}_{t-1}, \mathbf{u}_t) \\ \mathbf{y}_t &= \mathbf{h}_t(\mathbf{x}_t, \mathbf{v}_t) \end{aligned} \tag{1}$$

where $t \in \mathbb{N}$ is a discrete-time index, $\mathbf{x}_t \in \mathbb{R}^n$ is a state vector of the system at $t$, $\mathbf{y}_t \in \mathbb{R}^m$ is a vector of observations, and $\mathbf{u}_t \in \mathbb{R}^p$ and $\mathbf{v}_t \in \mathbb{R}^q$ are noise vectors. The mapping $\mathbf{f}_t : \mathbb{R}^n \times \mathbb{R}^p \mapsto \mathbb{R}^n$ is known as a system transition function, and $\mathbf{h}_t : \mathbb{R}^n \times \mathbb{R}^q \mapsto \mathbb{R}^m$ as a measurement function. The analytic forms of the two functions are assumed known. The first of the equations is usually referred to as state equation and the second, as observation equation.

There are three different classes of signal processing problems related to the model described by (1):

1. filtering: $\forall\, t$, estimate $\mathbf{x}_t$ based on $\mathbf{y}_{1:t}$,
2. prediction: $\forall\, t$ and some $k > 0$, estimate $\mathbf{x}_{t+k}$, based on $\mathbf{y}_{1:t}$, and
3. smoothing: for all $t$, estimate $\mathbf{x}_t$, based on $\mathbf{y}_{1:T}$, $t \in \mathbb{Z}_T = \{1, 2, \ldots, T\}$

where $\mathbf{y}_{1:t} = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_t\}$. Another very important objective is to carry out the estimation of the unknowns *recursively* in time.

A key expression for recursive implementation of the estimation is the update equation for the posterior density of $\mathbf{x}_{1:t} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_t\}$ given by

$$p(\mathbf{x}_{1:t}|\mathbf{y}_{1:t}) = \frac{p(\mathbf{y}_t|\mathbf{x}_t)\, p(\mathbf{x}_t|\mathbf{x}_{t-1})}{p(\mathbf{y}_t|\mathbf{y}_{1:t-1})} p(\mathbf{x}_{1:t-1}|\mathbf{y}_{1:t-1}). \tag{2}$$

Under the standard assumptions that $\mathbf{u}_t$ and $\mathbf{v}_t$ represent additive noise and are independently and identically distributed according to multivariate Gaussian distributions and that the functions $\mathbf{f}_t(\cdot)$ and $\mathbf{h}_t(\cdot)$ are linear in $\mathbf{x}_{t-1}$ and $\mathbf{x}_t$, respectively, the above problems are optimally resolved by the Kalman filter [1]. When the optimal solutions cannot be obtained analytically, one resorts to approximations of the posterior distributions [9], or the extended Kalman filter or Gaussian sum filters [1].

The set of methods known as particle filtering methods are based on a different paradigm. They approximate the posteriors by particles (samples) from the posteriors. If the particles $\mathbf{x}_t^{(i)}$, $i = 1, 2, \cdots, M$ have respective importance weights, $w_t^{(i)}$, they approximate the posterior and allow for computation of all sorts of estimates of the unknowns. As new data become available, the particles are propagated exploiting (2) and the concept of sequential importance sampling. A critical role in the application of the filter is played by the so called importance function, which is used for generation of the particles $\mathbf{x}_t^{(i)}$. The ideal importance function $\pi(\mathbf{x}_t|\mathbf{x}_{1:t-1}, \mathbf{y}_{1:t})$ is given by

$$\begin{aligned} \pi(\mathbf{x}_t|\mathbf{x}_{1:t-1}, \mathbf{y}_{1:t}) &= p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{y}_t) \\ &\propto p(\mathbf{y}_t|\mathbf{x}_t) p(\mathbf{x}_t|\mathbf{x}_{t-1}). \end{aligned} \tag{3}$$

Much of the activity in particle filtering in the sixties and seventies was in the field of automatic control. With the advancement of computer technology in the eighties and nineties, the work on particle filters intensified and many new contributions appeared in journal and conference papers. A good review article on the subject with most of the relevant references is [5].

## 2 Applications in signal processing

Here we list some important signal processing problems whose resolution with particle filters is of great interest.

**Positioning, target tracking, and navigation:** Positioning, target tracking, and navigation is of significance in various areas, including radar and sonar. The main objective there is to estimate recursively the hidden states of objects that may represent the positions of moving objects, their velocities, altitudes, and/or directions. For example, in tracking, the states represent the objects' positions, and the state equation (1) describes the dynamics of the objects. On the other hand, the observation equations model the measurements as functions of the coordinates of the objects. Papers that have addressed this problem with particle filters include [2, 7, 8]. In [7] a particle filter is proposed that uses $p(\mathbf{x}_t|\mathbf{x}_{t-1})$ as importance function and employs the concept of sampling-importance-resampling.

**Blind deconvolution:** Deconvolution is a signal processing problem of fundamental importance. It amounts to determining the input of a linear shift-invariant system from the observed output and the pulse response of the system. A much more difficult task is blind deconvolution, where the input of the system, that is, the transmitted signal which is received distorted by a receiver, has to be estimated without knowledge of the system's response. If the system is time varying, the task gets even more difficult. The last scenario is typical in mobile communications, where the communication channels (the states) are time varying and are known as flat fading channels. The objective is to detect the transmitted symbols and estimate the communication channel simultenously. Applications of particle filters to this problem can be found in [3, 10, 11]. A related task is channel equalization where the objective is to compensate for the distortions of the channel with the goal of minimizing the probability of detection error of the transmitted symbols.

**Recursive estimation with discounted measurements:** In adaptive signal processing the class of exponentially weighted recursive least-squares filters plays a major role. They are used in situations where the signals or systems vary in time in an unknown way and for which we do not have a model. With exponential weighting of the data, they attempt to track the dynamics of the signals or systems by weighting the observed data, emphasizing the current and discounting the old ones. The same idea can be implemented with particle filters, where a system equation representing a random walk is introduced, and the observation equation remains unchanged [4]. With the selection of noise parameters in the system equation, one can implement various intensities of discounting. This method is very attractive because it increases considerably the scenarios in which the discounted measurement idea can be implemented.

**Other applications:** Particle filters have found applications in other signal processing problems where on-line signal processing in nonlinear and/or non-Gaussian problems is required. An interesting example is the processing of time-varying autoregressive models with the objective of estimating time-varying spectra [6].

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