A General Index for the Evaluation of Gap between the Real and the Ideal Customer Satisfaction

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1. Index G_l

The satisfaction of customers of a service, or of purchasers of a good, is normally expressed with a score on a ordinal or cardinal scale. We assume that, for an informed customer, top and bottom levels of quality, given the social, economical and natural environment of service supply, do exist and are unique. Minimum and maximum levels of service quality differ according to individuals, but the offer of a common evaluative scale defines the extremes and attributes a common meaning to any intermediate position.

Let us suppose a sample on n customers is drawn from a population of N, and subject positions are measured with a common quantitative scale, of which m denotes its minimum and M its maximum values. The "average gap" between the real (i.e. the current) and the most satisfactory position may be expressed as a Minkowski distance of parameter λ:

$$G_l^* = \left[\frac{1}{n}\sum_{j=1}^{n}\left|y_j - M\right|^l\right]^{1/l} \qquad (l \geq 1)\,, \tag{1}$$

where y_j is the position of customers j (j=1, .,. n). If the scale is composed of k natural numbers, the standardised index, G_λ, is written as:

$$G_l = \frac{G_l^*}{Max(G_l^*)} = \frac{1}{M-m}\left[\sum_{i=1}^{k}\left|y_i - M\right|^l f_i\right]^{1/l} \qquad (l \geq 1)\,, \tag{2}$$

where n_i is the frequency of score y_i and $f_i = n_i / n$. G_λ is qualified by the following properties:

1. It varies between 0 and 1 $(0 \leq G_l \leq 1)$. It is null if all customers show full satisfaction, it attains its maximum if all customers are maximally critical toward the service, and , in between, it is sensible to any increase of satisfaction.
2. If l=1, the denominator of the index is the average of the absolute discrepancies from service optimality:

$$G_1 = \frac{\sum_{i=1}^{k}\left|y_i - M\right| f_i}{(M-m)} = \frac{\sum_{i=1}^{k}(M - y_i) f_i}{(M-m)} = \frac{M - \sum_{i=1}^{k} y_i f_i}{(M-m)}\,; \tag{3}$$

if l=2, the denominator is the average Euclidean distance from service optimum

$$G_2 = \left[\sum_{i=1}^{k}\left(y_i - M\right)^2 f \Big/ (M-m)^2\right]^{1/2} = \frac{1}{M-m}\left[\sum_{i}^{k}(y_i - M)^2 f_i\right]^{1/2}. \tag{4}$$

2. Distribution features of indexes G_1 and G_2

The expected value of index G_1 is:

$$E(G_1) = \frac{M - E(\bar{y})}{M-m} = \frac{M - m}{M-m} = 1 - \frac{m - m}{M-m}\,, \tag{5}$$

where $\bar{y}$ is the sample mean of Y, and m is the average satisfaction of the population of customers.

Variance of G_1 is proportional to the variance of satisfaction measures:

$$Var(G_1) = Var\frac{M - \sum y_i f_i}{M - m} = \frac{1}{(M-m)^2} Var(\bar{y}) = \frac{1}{(M-m)^2} \frac{S_y^2}{n}\left(1 - \frac{n}{N}\right) Deff \text{ ,} \qquad (6)$$

where $S_y^2 = \frac{1}{N-1}\sum_j (Y_j - m)^2$ is the elementary variance of Y, and $Deff$ is the so-called "design effect", whose formula depends on how the sample was formed (Kish, 1965).

The expected value of G_2, if the sample is simple random, is a bounded function of the average, m, and variance, s_y^2, of satisfaction levels of the population:

$$E(G_2) = \frac{E\left[s_y^2 + (\bar{y} - M)^2\right]^{1/2}}{M - m} \le \frac{\sqrt{2}}{M-m}\left(M - \frac{m}{2} - \frac{m^2 + s_y^2}{8M}\right), \qquad (7)$$

where $s_y^2 = \frac{1}{n-1}\sum_j (y_j - \bar{y})^2$ is the sample variance of Y, and the limit is obtained by expanding the analytic formula in a Taylor series.

Variance of G_2 is analogously bounded (Fabbris, 2000):

$$\frac{Var(\bar{y})}{(M-m)^2} \le Var(G_2) \le \frac{Var(\bar{y}) + m^2 + s^2/2 - M^2}{(M-m)^2} - \left[\frac{\left(m^2 + s^2\right)\left(m^2 + s^2 + 8mM\right)}{32M^2(M-m)^2}\right]. \qquad (8)$$

3. Final considerations

1. Index G_λ is applicable with any quantitative scale.
2. E(G_λ) is independent on the scale adopted to measure satisfaction. Nevertheless, the measure of the gap is more accurate if the number of explicit scale values is larger: Fabbris (2000) defines an algorithm for transforming a k-point scale into a k/2 or k/3 point scale.
3. The expected value of G_2 is larger than, or equal to, G_1 : $1 \geq E(G_2) \geq E(G_1) \geq 0$.
4. If we adopt G_1 as a measure of residual dissatisfaction with respect to an ideal service, we equally weigh each recorded gap. G_2 , instead, is much more severe than G_1 with episodes of serious dissatisfaction. For instance, while computing G_2 , a gap of 4 from optimum weighs four times a gap of 2 from optimum, whilst, for the computation of G_1 , a gap of 4 "democratically" weights twice as much a gap of 2. Hence, G_2 may be adopted if a policy of serious dissatisfaction removal is performed.

REFERENCES

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RESUME

On propose un index général pour la mesure de la satisfaction de la clientèle d'un service. L'index est présenté comme une distance de Minkowski des différences du maximum observées. La distribution statistique de l'index avec paramètres λ=1 et λ=2 est dérivée.