Asymptotics of the Matrix Langevin Distributions on Stiefel Manifolds

Yasuko Chikuse
Kagawa University, Faculty of Engineering
2217-20 Hayashi-cho
Takamatsu, Kagawa
761-0396, Japan.
chikuse@eng.kagawa-u.ac.jp

1. Introduction

The Stiefel manifold $V_{k,m}$ is the space a point of which is a set of k orthonormal vectors in $R^m(k \leq m)$, so that $V_{k,m} = \{X(m \times k); X'X = I_k\}$, where I_k is the $k \times k$ identity matrix. Special cases are the unit hypersphere $V_{1,m}$ of directed vectors and the orthogonal group $O(m) = V_{m,m}$ of $m \times m$ orthonormal matrices. The analysis of data on $V_{k,m}$ is required especially for $k \leq m \leq 3$ in practical applications in Geological Sciences, Astrophysics, Biology, Meteorology, Medicine and other fields.

The matrix Langevin $L(m, k; F)$ distribution is of exponential type whose density function is given by

$$\exp(tr\ F'X)/{}_0F_1(\frac{1}{2}m; \frac{1}{4}F'F),$$

for an $m \times k$ matrix F of rank $p(\leq k)$ with the singular value decomposition of F

$$F = \Gamma\Lambda\Theta',\ \text{where}\ \Gamma \in V_{p,m}, \Theta \in V_{p,k},\ \text{and}\ \Lambda = diag(\lambda_1, \ldots, \lambda_p),\ \lambda_1 \geq \cdots \geq \lambda_p > 0,$$

(see Downs (1972)). Corresponding to the normal distribution on the Euclidean space, the $L(m, k; F)$ distribution is most commonly used for statistical analyses on $V_{k,m}$. Some more discussions of the statistical analyses on $V_{k,m}$ and the $L(m, k; F)$ distribution may be found in e.g., Jupp and Mardia (1979), Watson (1983), Prentice (1986), and Chikuse (1998, 1999).

2. Asymptotics of the $L(m, k; F)$ Distribution

The distribution and inference problems concerning the $L(m, k; F)$ distributions involve hypergeometric functions ${}_pF_q$ with matrix arguments, the solutions of which seem to be intractable ; e.g., estimation and tests for hypotheses of the orientation parameters Γ and Θ and the concentration parameters Λ of $F = \Gamma\Lambda\Theta'$, classifications of matrix Langevin distributions, orthogonal associations (correlations), orientational regressions, and the related sampling distributions of various (matrix) statistics. These problems can be evaluated asymptotically for large sample size (and small concentrations), for high dimension m, and for large concentrations Λ.

Large sample asymptotic theory is concerned in connection with tests for uniformity ($\Lambda = 0$) of the $L(m, k; F)$ distribution, discussing asymptotic properties, near the uniformity, of the parameter estimation (of Γ, Θ and Λ) and of some optimal tests for uniformity (see Chikuse (1991)).

High dimensional asymptotic behaviour of some statistics is investigated, considering the problems of estimation and tests for hypotheses of parameters of the $L(m, k; F)$ distribution (see Chikuse (1993)).

Asymptotic theory is developed for the concentrated $L(m, k; F)$ distributions (i.e., for large Λ), discussing the estimation of large concentration parameters Λ, tests for hypotheses of the orientation parameters Γ and Θ, classifications of matrix Langevin distributions, measures of orthogonal association defined for random points on $V_{k,m}$, and orientational regressions (see e.g., Chikuse (2000)).

REFERENCES

Chikuse, Y. (1991). Asymptotic expansions for distributions of the large sample matrix resultant and related statistics on the Stiefel manifold. Journal of Multivariate Analysis 39, 270-283.

Chikuse, Y. (1993). High dimensional asymptotic expansions for the matrix Langevin distributions on the Stiefel manifold. Journal of Multivariate Analysis 44, 82-101.

Chikuse, Y. (1998). Density estimation on the Stiefel manifold. Journal of Multivariate Analysis 66, 188-206.

Chikuse, Y. (1999). Procrustes analysis on some special manifolds. Communications in Statistics- Theory and Methods 28, 885-903.

Chikuse, Y. (2000). Concentrated matrix Langevin distributions.

Downs, T.D. (1972). Orientation statistics. Biometrica 59, 665-676.

Jupp, P.E. and Mardia, K.V. (1979). Maximum likelihood estimators for the matrix von Mises-Fisher and Bingham distributions. Annals of Statistics 7, 599-606.

Prentice, M.J. (1986). Orientation statistics without parametric assumptions. Journal of Royal Statistical Society Series B 48, 214-222.

Watson, G.S. (1983). Statistics on Spheres. Lecture Note in Mathematics (University of Arkansas) 6, Wiley. New York.

RÉSUMÉ

Cet article considère les propriétés asymptotique de la distribution de Langevin définie dans le manifold Stiefel.