Testing for Stationarity in Panel Data

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1. Introduction

In this paper we extend Kwiatowsky *et al.* (1992) tests to panel data (for more information see Hadri (1999)). The tests we are proposing are shown to have an asymptotic normal distribution. The moments of our asymptotic distribution are calculated exactly.

2. The Tests

We consider the following two models:

$$y_{it} = r_{it} + \boldsymbol{e}_{it}, \qquad (1)$$

$$y_{it} = r_{it} + \boldsymbol{b}_i t + \boldsymbol{e}_{it}, \qquad (2)$$

where r_{it} is a random walk:

$$r_{it} = r_{it-1} + u_{it}.$$

Here y_{it}, $t = 1,..,T$ and $i = 1,..,N$ are the observed series for wich we wish to test stationarity for all i, the $\boldsymbol{e}_{it}$ and u_{it} are mutually independent and iid with $E\,[\boldsymbol{e}_{it}] = 0$, $E\,[\boldsymbol{e}_{it}^2] = \boldsymbol{s}_e^2 > 0$, $E[u_{it}] = 0$ and $E[\,u_{it}^2] = \boldsymbol{s}_u^2 \geq 0$. The initial values r_{i0} are treated as fixed unknowns and play the role of heterogeneous intercepts. Model 2 includes fixed effects and individual trends. The stationary hypothesis is simply $\boldsymbol{s}_u^2 = 0$. Since the $\boldsymbol{e}_{it}$'s are assumed iid, then under the null hypothesis y_{it} is stationary around a level in model 1 and trend-stationary in model 2. The test takes the following form:

$$H_0 : \boldsymbol{r} = \frac{\boldsymbol{s}_u^2}{\boldsymbol{s}_e^2} = 0, \quad \text{against } H_1 : \boldsymbol{r} > 0.$$

The LM (and LBI) statistic for panel data becomes:

$$LM = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} S_{it}^2}{\hat{\boldsymbol{s}}_e^2}, \qquad (3)$$

where S_{it} is the partial sum of the residuals and $\hat{\boldsymbol{s}}_e^2$ is a consistent estimator of $\hat{\boldsymbol{s}}_e^2$ under H_0.

3. Asymptotic Distribution of the Tests

In this section we consider the asymptotic distribution of the LM statistic given in (3) for each of the two models described above. All our limits use sequential asymptotic in which $T \to \infty$ followed by $N \to \infty$.

We show for model 1 that

$$L\hat{M}_m \to E\left[\int_0^1 V(r)^2\,dr\right] = \boldsymbol{x}_m, \qquad (4)$$

and that

$$Z_m = \frac{\sqrt{N}\left(L\hat{M}_m - x_m\right)}{V_m} \Rightarrow N(0,1), \qquad (5)$$

where $V(r)$ is the standard Brownian bridge $V(r) = W(r) - rW(r)$, $W(r)$ is a standard Wiener process and $V_m^2 = \mathrm{var}\left(\int V^2\right)$. To calculate x_m and V_m^2 we used the characteristic function technique which gave:

x_m = 1/6 and V_m^2 = 1/45.

The index m indicates that the residuals come from model 1.

For model (2) we obtain the residuals from the regression of *y* on an intercept and the time trend. We show that

$$L\hat{M}_t \to E\left[\int_0^1 V_2(r)^2\,dr\right] = x_t\,, \qquad (6)$$

and that

$$Z_t = \frac{\sqrt{N}\left(L\hat{M}_t - x_t\right)}{V_t} \Rightarrow N(0,1), \qquad (7)$$

where $V_2(2)$ is the so-called second level Brownian bridge and $V_t^2 = \mathrm{var}\left(\int V_2^2(r)\right)$. x_t and V_t^2 are computed as indicated above:

x_t = 1/15 and V_t^2 = 11/6300.

Remark 2. We can also relax the assumption on the errors e_{it} being iid over *t* to accommodate serial dependence cases. In this case we have to replace s_e^2 by the long run variance defined by:

$$s^2 = \frac{1}{N}\sum_{i=1}^{N}\lim_{T\to\infty} T^{-1}\left(S_{iT}^2\right).$$

5. Conclusions

In this paper, we propose statistical tests of the hypothesis of stationarity against the alternative of a unit root in panel data. The tests can be applied to cases where the disturbance terms are heteroscedastic and serially correlated. Monte Carlo simulations show that the tests have good small sample properties.

REFERENCE

Hadri, K. (1999) "Testing the null hypothesis of stationarity against the alternative of a unit root in panel data with serially correlated errors," Research paper No. 9905, Dpt. of Economics, The University of Liverpool.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992) "Testing the null hypothesis of stationarity against the alternative of a unit root," *Journal of Econometrics*, 54, 91-115.

RESUME

This paper proposes a residual based Lagrange-Multiplier (LM) tests for a null that the individual observed series are stationary around a deterministic level or around a deterministic trend against the alternative of a unit root in panel data. The asymptotic distributions of the statistics are derived under the null and are shown to be normally distributed. Finite sample sizes and powers are considered in a Monte-Carlo experiment. The empirical sizes of the tests are close to the true size even in small samples. It is shown how to apply the tests in the interesting case where the disturbance terms are heteroscedastic and serially correlated