Variance Components with Generalized Symmetric Sums Biased/Unbiased Estimates

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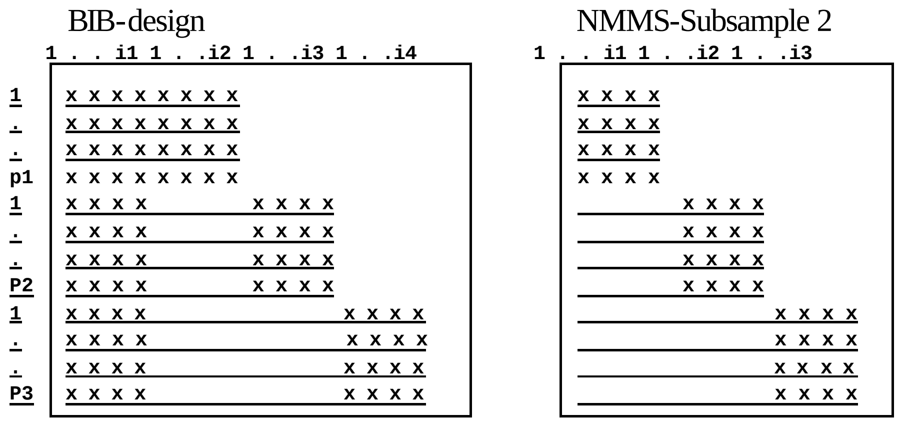
1. Matrix sampling and variance components

In measurement theory, variance components are the basis for the generalizability theory, which is used to estimate the sufficient size of an item test. What makes the situation complex is that the collected sample matrices are not rectangular, usually, but different designs containing a lot of structural missing values. Those kind of matrices are, for instance, S-, L- and +-designs and some more complex as BIB- and NMMS-designs. In the NMMS (not overlapping matrix designs) different rotations between booklets have been tested. In general, these sampling designs are called a matrix sampling designs.



There are several ways to calculate variance components, but usually missing values are not allowed or they must follow clear pattern. Previously, in ISI meetings 1993 and 1995, I have presented a solution for the model suggested by Sirotnik & Wellington (1977), in which the matrix form is not bound to any fixed matrix form, but the missing values could be in any of the sampling matrix cells not following any defined rule. The estimate formulae are given by above authors, Sirotnik & Wellington, but the error terms as well as the practical solution is given by the author of this paper (Tormakangas, 1997).

The estimation method is based on Generalized Symmetric Sums (GSS), in which the variance components depend on the linear combinations of the GSS's. The method with practical solution in case of different sample matrices is presented in Tormakangas, 1997.

2. Unbiased/Biased estimates

In common sample matrix there is usually an equal number of items per a student and in this case the symmetry exists naturally. The most of the tested sample matrices were this type or there was equal number of items in the common test for all students or in rotated booklets for subgroups of students and the components and error variances calculated for these matrices led to the assumption that this method would be unbiased.

Sometimes, however, in a large set of tests, the program resulted negative error variance estimates of the variance components and this happened, most commonly, for the item component. In those cases, examining the variance components more closely, it was also noted that the estimates calculated for these submatrices had clearly different variance component estimates than were the corresponding estimates for the original sample matrix. The difference was larger than what could be treated as an estimation error and thus the assumption was made that the components were biased.

By examining the matrices, in which the difference of estimates occurred more closely, it was noted that there was a different number of items for different student groups i.e. in testing situation, there was a different number of items in different test booklets. Obviously this is a property, which is in the contradiction with the symmetry assumption of a generalized symmetric sums and this led to the assumption that the estimation method is a general solution only for matrices, which satisfies the symmetry assumption meaning that **each student should have an equal number of items in the sample matrix**. The location of items on the observation row is not restricted.

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