Estimation of bilinear time series models

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Abstract : The present paper gives another approach of parametric estimation of the bilinear time series models. We construct the estimates of the coefficients of a biliear model using the minimum Hellinger distance (MHD) method. Under some probabilistic properties and other mild assumptions, we establish the consistency and the asymptotic normality of the MHD estimates.

The bilinear time series model which we consider is defined as follows :

$$X_t = \sum_{i=1}^{k} \phi_i X_{t-i} + \sum_{j=1}^{l} \pi_j e_{t-j} + \sum_{i=1}^{r} \sum_{j=1}^{s} b_{ij} X_{t-i} e_{t-j} + e_t \qquad (1.1)$$

where $(e_t; t = 0, \pm 1, ...)$ is a sequence of independent and identically random variables, with zero mean and finite variance σ^2. The distribution of e_t is assumed to be known. $\phi_i, (1 \leq i \leq k)$, $\pi_j, (1 \leq j \leq l)$ and $b_{ij}, (1 \leq i \leq r, 1 \leq j \leq s)$ are the parameters to be estimated.

Many applications of this class of non-linear time series models exist in Tong [1]. The probabilistic properties of the model are well known (see Liu [2], Liu et Brockwell [3] and Pham [4]). The purpose of this paper is to estimate, by the minimum Hellinger distance (MHD) method, the coefficients of the model (1.1) and to establish the consistency and the asymptotic normality of the MHD estimates. The advantage of this method of parametric estimation is that the MHD estimates are robust under perturbations (see. Beran [5] and Hili [6]). Furthermore, there is no restriction in the shame of the model in (1.1) when applying the MHD method.

We proceed as follows. Let denote by θ the parameter vector
$\theta = (\phi_1, ..., \phi_k, b_{11}, ..., b_{rs}, \pi_1, ..., \pi_l)'$ which is assumed to describe a compact set Θ of $\mathbb{R}^{k+l+rs}$. The prime denotes the transpose. We first recall the assumptions which ensure that (X_t) is a second order stationary strongly mixing process for which the marginal density f_θ is known up to θ. Then, given N observations X_1,...,X_N for which the true commun density is f_{θ_0}, we determine a MHD estimate of θ_0 which is based on a good approximation of f_{θ_0} with respect to the familly $(f_\theta,\ \theta \in \Theta)$.
Let denote by $\hat{f}_N$ the kernel density estimate of f_{θ_0}. Then

$$\hat{f}_N(u) = (1/N)\sum_{j=1}^{N}(1/b_N)K\left[(u - X_t)/b_N\right], \quad u \in \mathbb{R}$$

where (b_N) is a sequence of banwidth and K is a kernel function. The MHD estmate $\hat{\theta}_N$ of θ is :

$$\hat{\theta}_N = \text{Arg}\min_{\theta\in\Theta}\left\{\int_{\mathbb{R}}|\hat{f}_N^{1/2}(u) - f_\theta^{1/2}(u)|^2 du\right\}^{1/2}.$$

Under some other assumptions, we show that $\hat{\theta}_N$ is a consistent estimate and asymptotically admits a normal distribution.

References :

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