Shrinkage interval estimator for the exponential mean lifetime

Smail Mahdi

University of the West Indies, Department of Computer Science, Mathematics and Physics

Cave Hill Campus, PO Box 64, Bridgetown, Barbados

E-Mail smahdi@uwichill.edu.bb

1. Introduction

Although preliminary test point and interval estimators initiated in Bancroft (1944) often provide better results than unconditional estimators, they have been found to possess high risk. Therefore, shrinkage versions of preliminary test estimators which dominate in terms of bias and risks the usual preliminary test estimators have been recently proposed. Accordingly, we propose the following shrinkage interval estimator for the population exponential mean lifetime

$$I^S = \begin{cases} I_1 & \text{if } \eta \in \mathcal{D} \\ \\ I_2^S & \text{if } \eta \notin \mathcal{D} \end{cases} \tag{1}$$

where

$$I_2^S = \begin{cases} I_1 & \text{with probability } \gamma \\ \\ I_2 & \text{with probability } 1-\gamma. \end{cases} \tag{2}$$

I^S is a double mixture of the usual unconditional interval I_1 and the pre-test conditional interval I which is a mixture of I_1 and I_2 with respective probabilities $Pr(\eta \in \mathcal{D})$ and $1 - Pr(\eta \in \mathcal{D})$ (see Mahdi (1999)). The shrinkage estimator relies on the test of a null hypothesis H_0 based on the outcome of a statistic η and a Bernoulli outcome with probability of success γ. $\mathcal{D}$ is the critical region for H_0. The probability $1-\gamma$ reflects the degree of confidence in H_0 when this hypothesis is not rejected. However, the prior choice of γ is very ambiguous and the experimenter must often rely on data at hand to fix γ. Note that for $\gamma = 0$, $I^S = I$ and for $\gamma = 1$, $I^S = I_1$. In the absence of any prior information, we propose to use $\gamma = 1 - Pv(\eta)$ where $Pv(\eta)$ is the P-value corresponding to the observed η. Therefore, the more $Pv(\eta)$ is large, the more we use $I_2^S = I_2$ as it should be. An analytical study and a numerical investigation of the performance of the proposed interval I^S with respect to interval I and I_1 will be presented. Simulation results for the cases $\gamma = 0(.1)1$ when the pre-test is performed with the optimal significance level $\epsilon = 0.0001$ for I are displayed in Table 1.

2. Coverage probability and expected length of the shrinkage interval I^S

The coverage probability of the shrinkage interval I^S is given by

$$CP(I^S) = CP(I) + \gamma[CP(I_1) - CP(I)]. \quad (3)$$

Similarly, the expected length of the shrinkage interval I^S is given by

$$EL(I^S) = EL(I) + \gamma[EL(I_1) - EL(I)]. \quad (4)$$

If $\gamma = 0$, then $CP(I^S) = CP(I)$ and $EL(I^S) = EL(I)$ as it should be. Furthermore, for $\gamma = 1$, we have $CP(I^S) = CP(I_1)$ and $EL(I^S) = EL(I_1)$ as it should be as well.

3. Simulation and discussion

Simulation results displayed in table 1 below show that the overall performance of interval I^S is intermediate to those of I and I_1. It is then worth considering shrinkage intervals.

Table 1. Ratios of average coverage probabilities and average lengths of I^S over I_1 in the cases $\epsilon = .0001$, $5 \leq n_1 = n_2 \leq 25$, $0 < \phi \leq 1$ and $0 \leq \gamma \leq 1$.

γ	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$\phi \leq 0.5$											
$\overline{PCI^S}/\overline{PCI_1}$	.71	.74	.77	.80	.83	.85	.88	.91	.94	.97	1
$\overline{LI^S}/\overline{LI_1}$	.74	.77	.79	.82	.84	.87	.90	.92	.95	.97	1
$\phi \geq 0.5$											
$\overline{PCI^S}/\overline{PCI_1}$	.93	.94	.94	.95	.96	.97	.97	.98	.99	.99	1
$\overline{LI^S}/\overline{LI_1}$	.60	.64	.68	.72	.76	.80	.84	.88	.92	.96	1
$0 < \phi \leq 1.$											
$\overline{PCI^S}/\overline{PCI_1}$	.82	.84	.86	.87	.89	.91	.93	.95	.96	.98	1
$\overline{LI^S}/\overline{LI_1}$	.68	.71	.73	.77	.80	.84	.87	.90	.93	.97	1

REFERENCES

Mahdi, S. (1999). Monte Carlo studies on the accuracy of an interval estimator after a preliminary test of significance procedure. *Bulletin of the International Statistical Institute, 52nd session, Book* **2**, 253-254.

Bancroft, T.A. (1944). On biases in estimation due to the use of preliminary tests of significance. *Annals of Mathematical Statistics*, **15**, 190-204.

RÉSUMÉ

Un intervalle de confiance conditionnel ainsi qu'un intervalle de confiance rétréci pour l'estimation, de la moyenne d'une loi exponentielle, basée sur deux echantillons indépendants, l'un relatif à la population d'intérêt et l'autre à une population d'intérêt suspect, sont proposés. La performance de ces deux intervalles par rapport à celle de l'intervalle classique, basé uniquement sur l'échantillon pris dans la population d'intérêt, est étudiée.