

A Linear Programming Model for an Optimal Basic Premium in Car Insurance

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1. Introduction

In car insurance, the calculus of the basic premium is fundamental, since it is obligatory for all insure. The actuary determines this premium on the basis of the frequency and the size of claims. Different theoretical principles can be used in the calculus of this premium (see, [1],[2]), like the expected value principle, the variance principle, the exponential principle, etc...

The Bayesian approach is a powerful tool for a mathematical treatment in the car insurance (see [2]). In this paper, we are motivated by the use of the linear programming to determine an optimal basic premium, for the Algerian car insurance system, where the vehicles park contains more than three millions of vehicles [3], distributed on 18 classes of risks. The main factors that gave this partition of the population in classes of risks, were the vehicle power, the kind, the use and the zone of circulation. For a commercial reason, the age of the driver and the age of the vehicle are considered as factors of overcharge of the premium. In the Algerian car insurance system, seven contract periods are considered.

In this article we propose to determine the basic premium, for every tariff class, through an approach of modelling by a linear program with constraints. The program permits to determine an optimal basic premium, for each period. Also, the proposed model can be to use as a simulator of the basic premium, for the help to the decision. Parameters of the simulation can be to determine on the basis of claims statistic data.

2. Mathematical model

In an economy of market, all enterprise looks for the elaboration of a competitive tariffs to sell its products and attracts the clientele. The car insurance is an activity where the insurer must take account of this competitiveness, to manage its portfolio. Therefore, it is necessary for the insurer to use the scientific methods to fix his tariff. This motivates us to propose the following linear programming model for a scientific calculus of the basic premium.

$$\left\{ \begin{array}{l} \text{Min}(Z) = \sum_{j \in H} C_j p_j \\ (1 - \mathbf{a}) \sum_{j \in H} n b_j p_j = \sum_{j \in H} S_j \\ \frac{p_j}{j} \leq \mathbf{b} \left(\frac{p_{j-1}}{j-1} \right), \quad j \in H - \{j_1\} \\ (1 - \mathbf{a}) \left(\frac{j}{j-1} p_{j-1} \right) \leq p_j \end{array} \right.$$

. $H = \{j_k\}_{k=1,2,\dots,L}$: Set of contract periods, for example : $J_L = 365 \text{ days}$.

. p_j : the premium to pay, for a period j

. \mathbf{a} et \mathbf{b} : parameters chosen by the insurer, who designates respectively the loading rate and the solidarity parameter ($0 \leq \mathbf{a}, \mathbf{b} < 1$)

. $n b_j$: number of claims observed over the period j

. S_j : claim amount over the period j

. C_j : preference weights for each period j .

3. Computer implementation and software

Joined to a data base, the model is implemented as software of calculation and simulation for the basic premium and for the determination of bonus–malus premiums, based on a discrete Markov chain model, where the set of states are formed of tariff classes and the transitions probabilities can be estimated from the available information in the data base.

4. Conclusion

The proposed model is a scientific tool for an optimal calculation of the basic premium in the car insurance. The software that we have developed, on the basic of the model, is a tool of help to the decision for the insurer. The model is joined to a module of bonus-malus system, what constitutes a efficiency tool of prevention, against accidents of the road.

REFERENCES

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RESUME

Un programme linéaire , avec contraintes pour la modélisation de la prime de base en assurance automobile, est proposé. La fonction objectif minimise la somme des primes pondérées, sous des

contraintes d'équilibre économique et de limitation de la prime, fonction de la durée du contrat. Un logiciel interactif pour le calcul optimal de la prime est développé. Un système de bonus-malus est considéré