A procedure on estimating spectra of certain harmonizable processes

A. R. Soltani
Kuwait University and Shiraz University

Let us introduce second order processes $X(t),\ t \in Z$, that admit the following spectral characterization, namely,

$$X(t) = \int_0^{2\pi} g(x,t)\Lambda(dx), \quad t \in Z,$$

$$g(x,t) = \int_0^{2\pi} e^{ity} h(x,dy), \quad t \in Z$$

where $h(x,A)$ is a kernel on $[0,2\pi) \times [0,2\pi)$ and Λ is a second order process with independent increments, on $[0,2\pi)$. For simplicity only, the time t is taken to be discrete. Similar formulation can be set for continuous time. Also note that this formulation is different from the notion of time dependent spectra due to Priestley. Indeed the process $X(t)$, given above, is harmonizable:

$$X(t) = \int_0^{2\pi} e^{ity}\Phi(dy), \quad t \in Z,$$

$$\Phi(A) = \int_0^{2\pi} h(x,A)\Lambda(dx), \quad A \subset Z.$$

The random process Φ has, in general, dependent increments, and therefore the process $X(t)$, in general, is not stationary. Here are some examples.
Example 1. If the kernel $h(x,A)$ is atomic and has atom only on point x, then $X(t)$ is stationary.
Example 2. If the kernel $h(x,A)$ is atomic and has atoms on the d points $x + \frac{2\pi k}{d}$ in $[0,2\pi)$, then $X(t)$ is cyclostationary (or periodically correlated).

Example 3. If the kernel $h(x, A)$ is atomic and has atoms on $T_jT_k^{-1}(x),\ x \in B_k,\ \ k, j = 1, \cdots, m$, then X(t) is called simple, introduced by Soltani and Parvardeh, and Φ is given by $\Phi(A) = \sum_{j=1}^m \Psi_j(T_j^{-1}(A \cap B_j))$, where $B_1, \cdots, B_m$ is a partition for $[0, 2\pi)$, $T_j : B_1 \to B_j,\ \ j = 1, \cdots, m$ and $(\Psi_1, \cdots, \Psi_m)$ is a m-variate stationary process on B_1.
As we see the class of such non-stationary processes $X(t)$ is rich. It can explain certain non-stationary phenomena. The estimation of the spectral exponents, $g(t, y),\ f(dy) = E|\Lambda(dy)|^2$, based on the observations $X_1, \cdots, X_N$, is a challenging problem.
We propose, and sketch, the following procedure is. The key is to form

$$G_t^N(\lambda_p) = \sum_{q=0}^{N-1} c_{p,q} e^{i\lambda_{p,q} t}\ ,\ \ p = 0, \cdots . N-1,\ \ t \in Z,$$

where $c_{p,q}$ are certain constants and $\lambda_{p,q}$ runs through Fourier frequencies $\{\frac{2\pi j}{N},\ j = 0, \cdots, N-1\}$. When G_t^N is formed then the process $X(t)$ can be estimated by

$$\tilde{X}_t = N^{-1/2} \sum_{p=0}^{N-1} G_t^N(\lambda_p) d_Y(\lambda_p), \quad t \in Z,$$

where $Y_0, \cdots, Y_N$ is a finite segment of a certain discrete time stationary process, and $d_Y(\lambda_p)$ is the finite Fourier transform of the segment,

$$d_Y(\lambda_p) = N^{-1/2} \sum_{t=0}^{N-1} Y_t e^{-it\lambda_p}, \quad p = 0, \cdots, N-1,$$

By using $\tilde{X}_t$, a linear transformation will be established between $d_{\tilde{X}}$ and d_Y with coefficients $c_{p,q}$ that would lead to the estimation of the spectral density density f.

References

Soltani, A. R. (2000). A characterization theorem for stable random measures. Stochastic Anal. Appl. 18, 299-308.
Soltani, A. R. and Parvardeh, A. (2000). Simple random measures and simple processes. Preprint, Department of Statistics, Shiraz University.