Multilevel Models: School Effectiveness Studies Applications

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1. Introduction

Today an important problem in educational research is the assessment of school effectiveness. In statistical terms the problem is to describe and measure the relationship between one or more outcome and input variables of the educational process. The outcome variables are the examination results, attendance, disciplinary problems, etc., the input variables are related or to the characteristic of the pupil intake, like demographic and socio-economic background, sex, ability, etc, or to the school and teacher process, like class size, pupil-staff ratio, resources, etc.

In order to reach a conclusion about the importance of explanatory variables and the estimated effectiveness of individual schools, the general framework for analysis are multilevel models with outcome regressed on pupil intake, school and teacher variables. The use of such models is particularly appropriate because of the hierarchical structure of schooling systems. We refer to a hierarchy as consisting of units grouped at different levels. Thus, pupils may be the level 1 units grouped within schools that are the level 2 units. Educational research is interested in comparing school in terms of the achievements of their pupils.

The advantages of an analysis that explicitly models the manner in which pupils are grouped are related to the statistical treatment of clustered data, that concern the lack of independence of observations, the properties of the estimates and the fallacies related to aggregation or disaggregation of data. This last aspect in educational studies has caused much debate about the so-called “unit of analysis”, problem that we have if hierarchical structure is ignored.

The aim of this paper is to expose the reasons that make multilevel analysis suitable to solve the outlined problem.

2. The basic linear multilevel model

The equation of the basic linear 2-level model is the follow:

\[ Y_{ij} = (\beta X)_{ij} + u_j + e_{ij} \]  

(1)

Where \( Y_{ij} \) is the dependent variable where \( j \) refers to the level 2 unit and \( i \) to the level 1 unit, \( X \) is design matrix for the explanatory variable of the fixed part of the model, \( \beta \) is the vector of regression coefficients, \( u \) and \( e \) are independent random variables that represent, respectively, the residuals at group and at the individual level. This model includes only the random parameters \( \sigma^2_u \) and \( \sigma^2_\varepsilon \) and is termed “variance components model” because the variance of the response, about the fixed component, the fixed predictor, is

\[
\text{var}(Y_{ij} \mid \beta_0, \beta_1, x_{ij}) = \text{var}(u_0 + e_{oij}) = \sigma^2_u + \sigma^2_\varepsilon
\]

that is the sum of level 1 and level 2 variance. This model implies that the total variance for each pupil is constant and the covariance between two pupils in the same school is

\[
\text{cov}(u_{oij} + e_{oij1}, u_{oij} + e_{oij2}) = \text{cov}(u_{oij}, u_{oij}) = \sigma^2_u
\]

as level 1 residuals are assumed to be independent. The correlation between two students that is termed “intra level 2 unit correlation” is given by

\[
\rho = \frac{\sigma^2_u}{\sigma^2_u + \sigma^2_\varepsilon}
\]
It measures the proportion of the total variance, which is between schools. Multilevel models allow the comparison in the efficacies of schools that offer the same service to the pupils. The relative efficacy according to the definition of Willms includes both the effects of the management of the schools, and the effects of the social, economic and contextual factors on which schools haven’t any influence. The classification of the schools in terms of efficacies is given by the \( u \). According to the considered model, they assume several meanings that allow to identify the contribute of each school to the individual results of the pupils: a positive value indicates a performance greater than the mean, vice versa a negative value.

We illustrate the meaning of the residual terms \( u \) referring to the introduced basic model for a two levels analysis where it is supposed that the intercept only is variable between the groups. In this case the regression lines are parallels and their order is defined by the values of the terms \( u \). In order to express the efficacies of school according to the Willms classification, it is necessary to consider the structure of the design matrix that contains the values of the explanatory variables. Then, if \( X_{ij} \) includes individual characteristics only, the term \( u \) measure the “type A” efficacy; if \( X_{ij} \) includes also context variables that are not controlled by the schools, the terms \( u \) measure the “type B” efficacy. But, the design matrix is expressed in explanatory variables that could show a small amount of difference between the individual characteristics and the context variables, and these could have few distance from the management process variables too. For these reasons the “type A” effect may be well understood if we express separately the factors under the school control, that are the function of production of the schools (that are the resources, the structures of the organization), \( z \), from those related to the management whose factors don’t enter in the schools influence, \( u \) (Golstein e Thomas, 1996). With reference to the meaning that results may have for the interested subject, the ranking of the estimates of \( u \) give the ranking of schools, and an analysis of “type A” efficacy may solve the student problem of choice of the best school that can give the required education, an analysis of “type B” efficacy allows to choice the best management to which are interested in the controllers and the managers.

3. REFERENCES

RESUME

Ce travail regarde l’application de l’analyse multilevel pour l’étude de l’efficacité de l’école publique, en particulier on traite de la nature hiérarchique des donnés et de les deux types d’efficacité: efficacité de « type A » à laquelle sont intéressé les étudiants et les familles, et de l’efficacité de « type B » à laquelle sont intéressés les organes de contre ou d’administration ou les écoles même. L’application du modèle multilevel est apte à graduere les écoles et à permettre de faire l’analyse de leur comparaison..