A Generalization of a Stochastic Limit Theorem on a Non-Linear Stochastic Process

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A generalization of a spectral limit theorem on a non-linear stochastic process, with non-additive, independent, linear components (Venkataraman, [1]), is stated and proved in this paper. The validity of this generalization is discussed.

Introduction

Suppose that, for the integral $t, e_i(t); i = 1,2,\ldots,k$ are independent random variables satisfying the following basic assumptions.

Assumption 1: $E(e_i(t)) = 0$, and $E(e^4{}_i(t))$ is uniformly bounded with respect to $t; i = 1,2,\ldots,k$

Assumption 2 For $t \geq 1, E(e_i^2(t)) = s_i^2$, (say), is positive, and finite for $i = 1,\ldots,k$ (1)

Note: In this paper $E(\bullet)$ represents mathematical expectation.

Choose real numbers $l_i(r); r = 0,1,\ldots, \quad i = 1,\ldots,k$ such that

$$0 < |l_i(0)| + \sum_{r=1}^{\infty} r^{h_0} |l_i(r)| < \infty \tag{2}$$

for some real $h_0 \geq 1/2$

Define that,

$$Y(t) = Y_1(t)\ldots Y_2(t);$$

$$Y_i(t) = \sum_{r=0}^{\infty} l_i(t) l_i(t-r); \quad i = 1,\ldots,k \qquad t - \text{integral} \tag{3}$$

This paper is essentially concerned with the stochastic process $(X(t); t \geq 1)$, specified by the definition

$$X(t) = \sum_{r=0}^{\infty} a(r) Y(t-r); \quad t \geq 1 \tag{4}$$ where $a(r)$

are real, and satisfy the requirement that

Assumption 3:

$$0 < |a(0)| + \sum_{r=1}^{\infty} r^{h_0} | a(r) | < \infty \tag{5}$$

Note: All the random variables represented in this paper by infinite series, are understood to be limits in mean square of absolutely mean square convergent series.

The following notation is introduced towards describing the basic result of this paper.

$$\hat{f}_X(\theta) = (2\pi)^{-1} \sum_{r=-M}^{M} \cos\theta\, r . \hat{C}_X(r); \qquad -\pi < \theta \le \pi \tag{6}$$

where, (i) $\hat{C}_X(s) = N^{-1} \sum_{t=i}^{N-|s|} X(t) X(t-|s|); \quad |s| < N;$

(ii) M = integral part of N^{β}; $\quad (2h_0+1)^{-1} \le \beta < 1;$

(iii) $N \ge 2^{1/\beta}$ (so that $M \ge 2$) (7)

and, $$f_X(\theta) = \left| \sum_{p=0}^{\infty} \pi \exp(i\theta p) \right|^2 f_Y(\theta) > 0; \qquad -\pi < \theta \le \pi$$

where (i) $$f_Y(\theta) = (2\pi)^{-1} \left(\sigma_1^2 \ldots \sigma_k^2\right) \sum_{r=-\infty}^{\infty} \cos(\theta r)\, Q(r) > 0 \tag{8}$$

(ii) $$Q(r) = \prod_{i=1}^{k} \left(\sum_{u=0}^{\infty} \lambda_i(u) \lambda_i(u+|r|) \right) \tag{9}$$

Under this set up, the basic result of this paper is described below:

Theorem : *Chose distinct* $\theta_i; i = 1, \ldots, T$ in $(-\pi < \theta \le \pi)$, *such that for* $i \ne j, \theta_i + \theta_j \ne 0$. *Define further that* $\delta_i = \sqrt{2},$ *or* 1, *according* $\theta_i = 0, \pi,$ *or not. Then under the basic assumptions on* $(X(t), \quad t \ge 1)$

$$(N/2M)^{1/2} \left(\hat{f}_X(\theta_i) - f_X(\theta_i) \right); \; i = 1, \ldots, T;$$

converges in distribution $\left(\xrightarrow{d}\right)$, *as* $N \to \infty$, *to a normal vector* $(\eta(\theta_i); i = 1, \ldots, T)$. *This vector has zero mean and a diagonal covariance matrix, with diagonal elements*

$$E\left(\eta^2(\theta_i)\right) = \delta_i^2 f_X^2(\theta_i); \; i = 1, \ldots, T;$$

This Theorem is generalization of earlier result by Venkataraman (Theorem 2 of [1]).

Reference

[1] Venkataraman, K.N(1977) *A spectral limit theorem on a non-linear stochastic process with non-linear components.* The Annals of Institute of Statistical Mathematics Vol. 29, No.1 A pp 101-118.

Resume

On donne et démontre une généralisation d=un théoréme spectral limite sur un processus non-linéaire, avec des composants non-additifs, indépendents et linéaires (établi par Venkatarman, [1]) La validité de cette généralisation est discutée.