On Improved Estimator of the Generalized Variance for Natural Exponential Families

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The present paper is devoted to unbiased and minimum variance estimators of the generalized variance (i.e., determinant of the covariance matrix), say $|\Sigma|$, for X distributed as a probability $P(m; F)$ in a natural exponential family F on $\mathbb{R}^d$ with mean m and covariance matrix $\Sigma := V_F(m)$ (see Barndorff-Nielsen, 1978, or Letac, 1992, for more details).

If μ is a generating measure of F, then for all integer $n > d$, we construct a positive measure ν_n on $\mathbb{R}^d$ verifying

$$L_{\nu_n}(\theta) = |k''_\mu(\theta)|(L_\mu(\theta))^n, \tag{1}$$

where L_μ denotes the Laplace transform of μ, namely

$$L_\mu(\theta) = \int_{\mathbb{R}^d} \exp(<\theta, x>)\mu(dx),$$

and where $k_\mu = \log(L_\mu)$. Then there exists $C_n : \mathbb{R}^d \to \mathbb{R}$ such that

$$\nu_n(dx) = C_n(x)\mu^{*n}(dx). \tag{2}$$

We show that if $X_1, \cdots, X_n$ is n-random variables i.i.d. as $P(m; F) \in F$, then for all $n > d$, the UMVU estimator of the generalized variance $|V_F(m)|$ is

$$T_n = C_n(n\overline{X}), \tag{3}$$

where C_n is defined in (??) and $\overline{X} = n^{-1}\sum_{i=1}^n X_i$.

This result provides a unification and general extension of those appeared in Kokonendji and Pommeret (2001), Kokonendji and Seshadri (1996), and Pommeret (1998). Several natural exponential families are investigated through explicit expression and variance of this estimator; namely, Poisson-Gaussian, multinomial, gamma-Gaussian (Casalis, 1996), inverse Gaussian and Wishart distributions.

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RESUME

Nous proposons une construction de l'estimateur sans biais de variance minimum de la variance généralisée dans le cadre des familles exponentielles naturelles. Nous donnons une écriture de l'estimateur dans le cas des familles des lois de Wishart, et des lois gaussiennes inverses. Nous obtenons également sa variance dans le cas des familles quadratiques simples.