Statistical Inference in a Hierarchical Growth Curve Model

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We consider a growth curve model with hierarchical types of within-individuals design matrices. This model is an extension of the usual growth curve model introduced by Potthoff and Roy (1964). From a practical view, we state here the extended growth curve model with two hierarchical structures. The model is the following: we observe an $N \times p$ random matrix Y whose N rows are independently distributed, each with a p-variate normal distribution having unknown positive definite covariance matrix Σ. The means of the elements of Y are assumed to be of the form

$$E(Y) = A_1 \Xi_1 X_1 + A_2 \Xi_2 X_{(2)} = [A_1\ A_2] \begin{bmatrix} \Xi_1 & O \\ \Xi_{21} & \Xi_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix},$$

where A_i are known $N \times k_i$ design matrices between-individual design matrices of ranks k_i (i=1,2), X_1 and $X_{(2)}$= $[X_1'\ X_2']'$ are $q_1 \times p$ and $q \times p$ ($q = q_1+q_2 \le p$) within-individuals design matrices with rank q_1 and q, respectively. The parameters $\Xi_1 = [\Xi_{11}]$ and $\Xi_2 = [\Xi_{21}\ \Xi_{22}]$ are $k_1 \times q_1$ and $k_2 \times q$ unknown matrices, respectively.

For this model, the following problems have been studied by several authors:

(1) Maximum likelihood estimators and their basic properties
(2) The derivation of the AIC criterion for the model selection
(3) Testing the significance of hierarchical structures
(4) Likelihood ratio test for the hypothesis $\Xi_{22} = O$
(5) Simultaneous confidence regions of Ξ_1, Ξ_2 based simplified estimators and maximum likelihood estimators

In this paper, we consider LR test for the hypothesis

$$C \begin{bmatrix} \Xi_{11} & O \\ \Xi_{21} & \Xi_{22} \end{bmatrix} D = O$$

for some $c \times k$ matrix C and $s \times q$ matrix D under the assumption $A_1'A_2$=O. In particular we restrict ourselves in the following two cases: (i) C = $[C_1\ O]$ and D = $[D_1'\ O]'$, then the above hypothesis is $C_1 \Xi_{11} D_1$ = O, and (ii) C = $[O\ C_2]$ and D = $[D_1'\ D_2']'$, this case is $C_1[\Xi_{21}\ \Xi_{22}]D$ = O. Moreover, in the second case we consider the case where D is the following matrix

$$D = \begin{bmatrix} I_{q_1} & O & O \\ O & I_{q_3} & O \end{bmatrix}',$$

where $q_3 \le q_2$. Considering appropriate orthogonal matrices H and B, we can transform Y to a canonical form $Z = H'YB$ such that $Z \sim N_{N,p}(E(Z), \ \Omega \otimes I_N)$ with

$$\text{(a) } E(Z) = \begin{bmatrix} \Theta_{11} & \Theta_{12} & O & O \\ \Theta_{21} & \Theta_{22} & O & O \\ \Theta_{31} & \Theta_{32} & \Theta_{33} & O \\ O & O & O & O \end{bmatrix}, \quad \text{(b) } E(Z) = \begin{bmatrix} \Theta_{11} & O & O \\ \Theta_{21} & \Theta_{22} & O \\ \Theta_{31} & \Theta_{32} & O \\ O & O & O \end{bmatrix}$$

and $\Omega = B\Sigma B'$, where $H = [H_1\ H_2\ H_3\ H_4] \in O(N)$ and $B = [B_1'\ B_2'\ B_3'\ B_4']' \in O(p)$: H_i $(N \times r_i)$, B_i $(q_i \times p)$: $r_1 = c_1$, $r_2 = k_1+k_2-c_1$, $r_3 = N - r_1 - r_2$. Then the null hypotheses for testing problem (i) and (ii) are equivalent to

$$\text{(i)'} \quad \Theta_{11} = O, \qquad \text{(ii)'} \quad \Theta_{2(12)} = [\Theta_{21}, \Theta_{22}] = O,$$

respectively. The likelihood ratio statistic for (i)' is given by

$$\text{(c)} \quad \Lambda = |S_e| / |S_e + S_h|$$

where $W = [Z_{41}\ Z_{42}\ Z_{43}\ Z_{44}]'[Z_{41}\ Z_{42}\ Z_{43}\ Z_{44}]$, $S_e = W_{22\cdot 34} = W_{22} - W_{2(34)} W_{(34)(34)}^{-1} W_{(34)2}$, $S_h = (Z_{12} - Z_{1(34)} W_{(34)(34)}^{-1} W_{(34)2})'(I + Z_{1(34)} W_{(34)(34)}^{-1} Z_{(34)1})^{-1}(Z_{12} - Z_{1(34)} W_{(34)(34)}^{-1} W_{(34)2})$. Furthermore, the null distribution of this Λ is the ordinary Λ- distribution. Note that Λ does not depend $Z_2 = [Z_{21}\ Z_{22}\ Z_{23}\ Z_{24}]$ and $Z_3 = [Z_{31}\ Z_{32}\ Z_{33}\ Z_{24}]$.

The likelihood ratio statistic for testing problem (ii)' is given by

$$\text{(d)} \quad \Lambda = \Lambda^{(1)} \Lambda^{(2)}$$

where $\Lambda^{(i)} = |S_e^{(i)}| / |S_e^{(i)} + S_h^{(i)}|$. The explicit expressions of $S_e^{(i)}$ and $S_h^{(i)}$ are omitted here. We note that $\Lambda^{(1)}$ and $\Lambda^{(2)}$ are the significant test statistics for Θ_{21} and Θ_{22}, respectively. Moreover, under the hypothesis, $\Lambda^{(1)}$ and $\Lambda^{(2)}$ are mutually independent and are distributed as Λ-distribution.

REFERENCES

Potthoff, R. F. and Roy, S. N. (1964). A generalized multivariate analysis of variance model useful especially for growth curve problems. Biometrika 51, 313－326.

FRENCH RÉSUMÉ

Dans cet article, il s'agit d'un modèle élargi à courbe de croissance avec des matrices-plan à l'intérieur des individus ayant deux hiérarchies. Nous déduisons une ratio de vraisemblance statistique pour l'hypothèse lineaire de paramètres.