Wavelet Solution for Integral Equations

K. Maleknejad
Iran University of Science and Technology, School of Mathematics
Narmak, P.O.Box 16844
Tehran, Iran
e-mail: maleknejad@iust.ac.ir

H. Mesgarani
Iran University of Science and Technology, School of Mathematics
Narmak, P.O.Box 16844
Tehran, Iran
e-mail: mesgarani *@iust.ac.ir*

1.Wavelet Solution

The wavelet method[Mallat,S.G.(1988)] entails representing the solution u and the right hand side f as expansions of scaling functions at a particular scale m. For the proposes of the current work, it will suffice to say that the scaling function $\ddot{o}$ is defined by a dilation equation of the form

$$(1) \qquad j(x) = \sum_{k=-\infty}^{+\infty} a_k j(2x-k)$$

and that the values of the scaling function may be calculated using this recursion. Compactly supported scaling functions, such as those belonging to the Daubechies family[Daubechies,I.(1988)] of wavelets, have a finite number of non zero filter coefficients

a_k and the number of non zero coefficien ts is N.

We consider a linear Fredholm Integral equation of the second kind[Atkin son, Kendall.(1 997)] is an expression of the form

$$(2) \qquad u(x) - \int_a^b k(x,t)u(t)dt = f(x)$$

we use symbol K to denote the integral operator of equation (2) which is given by the formula

$$(3) \qquad (Ku)(x) = \int_a^b k(x,t)u(t)dt$$

for all $u \in L^2[a,b], x \in [a,b]$ and $k \in L^2([a,b]\times[a,b])$.Then equation (2) written in operator form is

$$(4) \qquad (I-K)u = f$$

The wavelet approximation to the solution *u(x)* at scale m is

$$u(x) = \sum_k \tilde{c}_k 2^{m/2} j(2^m x - k) \qquad k \in Z$$

$\tilde{c}_k$ is the wavelet coefficien ts of u, i.e. they define the solution in wavelet space. We make the substituti on $y = 2^m x$ then we may write

(5) $$u(x) = \sum_k c_k \boldsymbol{j}(y-k),\ k \in Z,\ c_k = \tilde{c}_k 2^{m/2}$$

Similarly, the wavelet expansion for *f (x) and k(x, t)* are

(6) $$k(x,t) = \sum_k \sum_l c_{k,l} j\ (y-k) j\ (s-l),\ k, l \in Z,\ c_{k,l} = 2^m \tilde{c}_{k,l}$$

$$f(x) = \sum_k g_k j\ (y-k), k \in Z, g_k = \tilde{g}_k 2^{m/2}$$

where $s = 2^{m/2} t.$

If we now substitute the expansions of *u(x), f(x)* and *k(x, t)* into our original Integral equation we have

(7) $$\sum_k c_k \boldsymbol{j}(y-k) - 2^{-m} \sum_k \boldsymbol{j}(y-k) \sum_l c_{k,l} c_l = \sum_k g_k \boldsymbol{j}(y-k)$$

Taking the inner product of both sides with $j\ (y-j), j \in Z,$ gives

(8) $$\sum_k c_k \int \boldsymbol{j}(y-k) \boldsymbol{j}(y-j) dy - 2^{-m} \sum_k \int \boldsymbol{j}(y-k) \boldsymbol{j}(y-j) dy \sum_l c_{k,l} c_l = \sum_k g_k \int \boldsymbol{j}(y-k) \boldsymbol{j}(y-j) dy$$

and since the orthogonality of the translates of the scaling function implies that

(9) $$\int j\ (y-k) j\ (y-j) dy = \boldsymbol{d}_{jk}$$

we may write

(10) $$c_j - 2^{-m} \sum_l c_{j,l} c_l = g_j$$

This completes the solution of *u*.

REFERENCES

Daubechies, I. (1988) Orthonormal bases of compactly supported wavelets, Comm. Pure Appl. Math, 41, 909-996.

Mallat, S. G. (1988)A theory for multiresolution signal decomposition the wavelet repressentation, Comm. Pure Appl. Math, 41, 674-693.

Atkinson, Kendall. (1997) The Numerical Solution of Integral equations of the second kind, Cambridge University press.

RESUME

Dans ces charges de la base du wavelet du papier résoudre l'approximatly,Fredholm équation intégrante du deuxième genre comme pour atteindre un système la solution ainsi sont utilisés de qui mènera à trouver le coefficient de la solution approximative de l'équation intégrante.