Solution of Integral Equation by Modify Conjugate Gradient Method

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1.Conjugate Gradient Method(CGM)

Most of Integral equations can be approximated to a linear operator equation:

$$Kf = g \qquad (1)$$

Where in general **k** is a kernel operator and **f** is unknown to be solved for a given **g**.
Numerical approach such as Moment method change the above operator equation to a Matrix equation

$$AX = Y \qquad (2)$$

When **A** is an **N**×**N** matrix(which the dimension **N** maybe more than 1000 for a large structure or complex system)and **X** is an unknown **N**×**1** vector for a given **N**×**1** vector **Y**.
One iterative algorithm Conjugate Gradient Method(CGM)is currently a focal point for much work in this area.
In (CGM)**v**' s are chosen as gradient of error function $\boldsymbol{f}$
Let

$$p_k = V_k \boldsymbol{a} \nabla \boldsymbol{f}_k$$
$$V_k = A' S r_k$$
$$P_0 = V_0$$
$$P_{k+1} = V_{k+1} + \boldsymbol{b}_k P_k$$

Where â is defined by assuming $P'_k SA$-orthogonal. After calculation

$$\boldsymbol{b}_k = -\frac{(HASr_k, P_k)}{(HP_k, P_k)}$$

Where **H** is defined as $\boldsymbol{f} = (r, Sr) = (e, He)$ which is based upon error criterion.
$\boldsymbol{a}_k$ Is chosen such that $\frac{\partial \boldsymbol{f}_k}{\partial k} \to 0$.

2.Modified Conjugate Gradient Method(MCGM)

Our modification is done on (CGM) with error function $\boldsymbol{f} = (r, r)$ which $S = I$ is an unity vector. To speed up the rate of convergence for all kind of matrices especially large ones, a special kind of weighting factor is used in each iteration.
Our results show that this method is suitable for all kind of matrices even ill-condition ones. This method will converge in theory and the rate of convergence for large order matrices is obviously faster than other methods. The algorithm of (MCGM) is shown below

Initial steps

Guess X_0

$$r_0 = Y - AX_0$$

$$\boldsymbol{b}_0 = \frac{1}{\|Ar_0\|^2}$$

$$P_0 = \boldsymbol{b}_0 Ar_0$$

Iterate k = **0,1,2,…**

$$\boldsymbol{a}_k = \frac{1}{\|AP_k\|^2}$$

$$X_{k+1} = X_k + \boldsymbol{a}_k P_k$$

$$r_{k+1} = r_k - \boldsymbol{a}_k AP_k$$

$$\boldsymbol{b}_{k+1} = \frac{1}{\|Ar_{k+1}\|^2}$$

$$P_{k+1} = P_k + \boldsymbol{b}_{k+1} Ar_{k+1}$$

Terminate if $\dfrac{\|r_{k+1}\|^2}{\|Y\|^2}$ is small.

Table shows results

Where A is Hilbert matrix of order 10 and B is [-5.000 -4.000 -3.000 -2.000 -1.000 0.000 1.000 2.000 3.000 4.000]

Test = 0.2664118E-07 N = 10

X(1) =	-0.6190392E+03	R(1) =	-0.6507302E-03
X(2) =	0.1529656E+04	R(2) =	0.7067793E-03
X(3) =	0.2704118E+04	R(3) =	0.6414499E-03
X(4) =	-0.4534412E+04	R(4) =	0.4412456E-03
X(5) =	-0.2046019E+04	R(5) =	0.8699128E-04
X(6) =	0.1906700E+04	R(6) =	-0.8409062E-04
X(7) =	-0.6663116E+03	R(7) =	-0.2264001E-03
X(8) =	-0.1607702E+04	R(8) =	-0.3962523E-03
X(9) =	0.1348999E+04	R(9) =	-0.4375436E-03
X(10) =	0.2438114E+04	R(10) =	-0.5663704E-03

References

Atkinson K.(1997),*The Numerical Solution of Integral Equations of The Second Kind*, publishing CAMBRIDGE University press, United Kingdom, pp. 291-303.

Greenbaum A. (1997), *Iterative Methods for Solving Linear Systems*, SIAM publications, Philadelphia.

Saad Y. (1996), *Iterative Methods for Sparse Linear Systems*, PWS Publishing, Boston. Wellesley, MA, USA.

Le résumé

Dans ce papier qui résout systèmes linéaires qui obtiennent d'équations intégrantes la procédure de l'iterative a basé sur les Modifié Conjuguez l'Inclinaison Method(MCGM) est considérez.

Exactitude et taux de la convergence plusieurs répètent et la méthode directe est considérez.