A New Approach to Distribution Fitting: Decision on Beliefs

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1. A Long Abstract

We introduce a new approach to distribution fitting which has been named Decision on Beliefs (DOB). The random variable X with unknown probability distribution function (PDF) f_X is given. In order to identify f_X from a set of candidate PDF's like $S=\{f_1, f_2, \ldots f_m\}$, we try to construct an algorithm with the greatest possible confidence. To achieve this purpose, we model this problem as a special case of Optimal Stopping Problem, in the fallowing manner. At each stage, a random observation is obtained from the PDF f_X and we can either make a decision to select one of the f_i's as f_X, or obtain another observation. We assume a cost C is attached to obtaining each observation and the total number of possible observations cannot exceed N.

If the total number of observation obtained at stage $i\ (i=1,2,\ldots,N)$ is equal to k, then vector $O_k=(x_1, x_2, \ldots, x_k)$ displays these observations. Since, making decision is done in a stochastic environment, so we can introduce a probability on the event $\{f_X \equiv f_i\}$. We call the probability of $\Pr\{f_X \equiv f_i\}$, the *belief* on PDF f_i. After obtaining each observation, the beliefs on PDFs are updated by using a formula which is derived from the Bayes theorem. So, this formula is used to calculate the posterior beliefs from the prior ones and we prove that this designed system of learning is convergant. That is, after getting enough observations and updating the beliefs, finally with probability one, the belief that observations come from its PDF converges to one and the other beliefs converge to zero.

After obtaining each observation and updating the beliefs, we find their order statistics by sorting them in ascending order. Then, by identifying the maximum belief at each stage, we either select its PDF for f_X or decide to continue the process by getting another observation according to the criterian that will be introduced later. So, at each stage, making decision is done on the basis of the maximum belief only we designate this maximum by f_{gr}. Therefore, the PDF which is finally chosen for f_X is if we can say, an MLE for it.

We divide the decision making space into $E_{i,j}$'s such that $E_{i,j}$ represents the subspace containing only two candidates for f_X which are f_i and f_j. Since we focus on the maximum belief at each stage, so we do our investigation just in $E_{i,gr}$'s subspaces. Hence, the number of subspaces reduces to $(m-1)$. The strategy of decision making in subspace $E_{i,gr}$ at stage n, is defined on a real value such as $d_{i,gr}(n)$. If the belief on f_{gr} at this subspace is equal to or greater than $d_{i,gr}(n)$, we choose it for f_X in $E_{i,gr}$ otherwise, we do not have any selection at this stage. By using stochastic dynamic programming an algorithm which maximizes the expectation of the probability of correct selection is derived. This algorithm gives us $d^*_{i,gr}(n)$, which is the amount of $d_{i,gr}(n)$ that maximizes the above probability. By adopting the MiniMax objective function, we define the minimum of these probabilities as the objective of total model.

So, we need to carry out the calculations in each of $(m-1)$ subspaces. However, by proving a theorem we show that we need to carry out the calculations only in one subspace which contains the two PDFs that have the greatest beliefs.

The typical DOB process is as follows:

Step 1: Observation: Get an observation from the unknown PDF f_X.

Step 2: Revising the Beliefs: Obtain the posterior beliefs from the prior beliefs and the new observation.

Step 3: Order Statistics: Sort the posterior beliefs and find the two that have the greatest amounts. Suppose that these two beliefs correspond to the PDFs f_{gr} and f_{sm}. Then, reduce the space of decision-making to $E_{sm,gr}$.

Step 4: Calculating d^*: Calculate the relevant value of $d^*_{sm,gr}(n)$.

Step 5: Decision: If the belief on PDF f_{gr} in subspace $E_{sm,gr}$, is equal to or greater than $d^*_{sm,gr}(n)$, then choose f_{gr} as f_X and the decision-making process will be ended. Otherwise, without having any selection at this stage, obtain another observation, lower the stage number to $(n-1)$ and return to step 1. The process will continue until we stop or the number of stages are finished.

To demonstrate how this process works, some numerical examples have been given in the full paper. Also a comparison between this method on one hand, and the K-S as well as the Chi square methods on the other is included. The results show that this DOB is so much more powerful than the above GOF methods.

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