A New Estimator for Ito-type Diffusion Process

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1. Introduction

We consider a vector-valued continuous-time stochastic process $\{x(t)\}$ in a state space $S \subset R^k$, generated by the following system of possibly non-stationary and/ or nonlinear Ito stochastic differential equations ;

$$dx(t) = a(t, x(t))dt + b(t, x(t))\theta dt + \sigma(t, x(t))dW(t)\ ,\ 0 \leq t \leq T \qquad (1.1)$$

where $x(t)^T = (x_1(t), \cdots, x_k(t))$, , $a(t, x)$, $b(t, x)$, $\sigma(t, x)$ are fixed $k \times 1$, $k \times p$, $k \times k$ matrix-valued functions respectively, $\theta^T = (\theta_1, \cdots, \theta_p)$ is an unknown vector of p parameters of interest with $p \leq k$, $W(t)^T = (W_1(t), \cdots, W_k(t))$ is a vector of k independent standard Brownian motion processes.

Recently , the use of continuous-time processes described by Ito stochastic differential equations has become very popular in such diverse fields as stochastic optimal control theory and financial economics. See Malliaris and Brock(1982) for an excellent survey of stochastic methods in economics and finance and Hull(1997) for more specific applications in various capital asset pricing models.

2. Aim

In this paper, we consider the parametric estimation problem of θ in the model (1.1) on the basis of the continuously observed values $\{x(t); t \in [0, T]\}$ of the process up to time T .

3. Methods

For the special model of the type (1.1) with $\sigma(t, x) = I_k\sigma$ for $\sigma > 0$, both conditional LSE and MLE of the unknown parameter θ are the same and is given by

$$\hat{\theta}_o = [\int_0^T b^T b(t, x(t))dt]^{-1} \int_0^T b^T(t, x(t))[dx(t) - a(t, x(t))dt].$$

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In spite of the elegant asymptotic optimality theory developed for this estimator , standard inference procedures based on LSE or MLE have several serious limitations from the practical viewpoint.

Thus we introduce the new estimator

$$\hat{\theta}_c = [\int_0^T K(t,x(t))b(t,x(t))dt]^{-1} \int_0^T K(t,x(t))[dx(t) - a(t,x(t))dt]. \qquad (1.3)$$

By the appropriate choice of the matrix $K(t,x(t))$, we can generate a variety of instrumental variables estimators of θ. For example LSE $\hat{\theta}_o$ corresponds to the choice of special instrumental variables $K(t,x(t)) = b^T(t,x(t))$.

4. Results

In this paper, we propose a special instrumental variables estimator and establish important finite sample properties of the estimator such as the exact median-unbiasedness and the exact normality of the corresponding pivotal quantity for the possibly non-stationary and/or nonlinear diffusion processes. Then we develop exact confidence intervals and exact tests of the hypotheses of θ valid for non-stationary and/or nonlinear diffusion models.

5. Conclusion

In this paper, we have proposed a new instrumental variable estimator of the parameter of the Ito type diffusion processes and developed exact confidence intervals and tests for the parameters which is valid not only for stationary but also possibly nonlinear and non-stationary processes.

Our approach to the parameter estimation is based on the special orthogonal instrumental variables of unit length and has a modest relative efficiency with respect to MLE for the stationary case. Furthermore it has many desirable small sample properties such as median-unbiasedness even for the possibly nonlinear and non stationary processes.

Reference

Hull, J.C. (1997). *Options, futures, and other derivatives*, 3-rd ed., Prentice-Hall International ed.

Malliaris, A.G. and Brock, W.A. (1982). *Stochastic methods in economics and finance*, North-Holland Publishing Company, New York.