Asymptotic Validity of Test Procedures for Linear Hypotheses of Split Mean Vectors in a General Linear Model

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１.Introduction

We consider a general linear model of the following form, for $n \geq 2$ and $p \geq 2$,

$$\underline{Y} = \begin{pmatrix} \underline{Y}_1 \\ \underline{Y}_2 \\ \vdots \\ \underline{Y}_n \end{pmatrix} = \begin{pmatrix} I_p & \underline{1}_p & \underline{0}_p & \cdots & \underline{0}_p \\ I_p & \underline{0}_p & \underline{1}_p & \cdots & \underline{0}_p \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I_p & \underline{0}_p & \underline{0}_p & \cdots & \underline{1}_p \end{pmatrix} \begin{pmatrix} \underline{\mu} \\ \underline{\tau} \end{pmatrix} + \begin{pmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \vdots \\ \underline{e}_n \end{pmatrix} \in R^{np} \qquad , \tag{1.1}$$

where $\underline{\mu} = (\mu_1, \mu_2, \ldots, \mu_p)' \in V_1 \subset R^p$ and $\underline{\tau} = (\tau_1, \tau_2, \ldots, \tau_n)' \in V_2 \subset R^n$ are unknown parameters, I_p is the identity matrix of order *p*, $\underline{1}_p$=(1,1,⋯,1)' and $\underline{0}_p$=(0,0,⋯,0)'$\in R^p$ and $\underline{e}_i$ are error vectors iid with $E\underline{e}_i = 0, E\underline{e}_i\underline{e}_i' = \Sigma$ where Σ is an unknown positive definite covariance matrix. Here, $V_1 = \{\underline{\mu} | \underline{\mu} \in R^p, \underline{\mu}'\underline{1}_p = 0\}$ with $\dim V_1 = p_1 = p - 1$ and V_2 is a subspace of R^n with $\dim V_2 = p_2 > 0$.

Arnold (1981, Sec. 10) showed the asymptotic validity of a test procedure usually given for a linear hypothesis about the mean vector in the ordinary linear model without the assumption of normality. Noda and Ono (2000), under a known covariance structure, constructed UMP invariant test procedures for linear hypotheses about the split mean vectors $\underline{\mu}$ and $\underline{\tau}$ in the model (1.1), respectively.

We here prove the asymptotic validity of the test procedures having the same forms as those in Noda and Ono (2000).That is, we show that the sizes of the tests under the null hypotheses are asymptotically unaffected by the underlying distribution and hence are the same as if the error vectors were distributed with multivariate normal distribution. Moreover the powers of these tests are proved to be asymptotically the same as those of UMP invariants as are given in Noda and Ono.

2. Main theorems

Let $\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_r$ be linearly independent and known vectors in V_1. We set a linear hypothesis,

$$H_{10} : \underline{c}_i'\underline{\mu} = 0 (i = 1,2,\ldots,r) \quad \text{against} \quad H_{11} : \underline{c}_i'\underline{\mu} \neq 0 \quad \text{for some } i. \tag{2.1}$$

Writing $C = (\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_r)$, we have a transformation $\underline{X}_i' = C'\underline{Y}_i (i = 1,2,\ldots,n)$ .Let $\underline{\overline{X}} = n^{-1}\sum_{i=1}^{n} \underline{X}_i$.

Theorem 2.1. Under the model (1.1) with $r < n-1$,

$$F_{1n} = n(n-r)r^{-1}\underline{\overline{X}}'[\sum_{i=1}^{n}(\underline{X}_i - \underline{\overline{X}})(\underline{X}_i - \underline{\overline{X}})']^{-1}\underline{\overline{X}} \to \chi_r^2(\delta_1)/r \quad \text{in distribution,} \tag{2.2}$$

as $n \to \infty$. Here δ_1 denotes the noncentrality as $\delta_1 = (C'\underline{\mu})'(C'\Sigma C)^{-1}(C'\underline{\mu})$ (2.3)

Using the statistic F_{1n}, we construct the following size α test,

$$\phi_{1n} = \begin{cases} 1 & rF_{1n} > \chi_r^{2,\alpha}, \\ 0 & rF_{1n} < \chi_r^{2,\alpha}, \end{cases} \tag{2.4}$$

To obtain the optimality of this test ϕ_{1n} ,we set the following assumptions.

(**M2**) There exists a (unknown) positive constant λ_0 such that $\Sigma\underline{1}_p = \lambda_0\underline{1}_p$.

(**M3**) The testing problem treated here is invariant under the group of affine transformations on R^{np} .

Theorem. 2.2. Under the assumptions (**M2**),(**M3**) with $r < n-1$, the statistic F_{1n} defined in (2.2) is asymptotically a maximal invariant under an affine transformation group and δ_1 defined in (2.3) is a parameter maximal invariant under the group induced by the one mentioned above, as $n \to \infty$.

By E_{δ_1} we denote the expectation with respect to the sampling distribution of F_{1n}. When $\underline{e}_i \sim N_p(\underline{0}_p, \Sigma)$, F_{1n}^* denotes the statistic defined in (2.2) and $E_{\delta_1}^*$ does the expectation with respect to the sampling distribution of F_{1n}^* . Then, as stated in Introduction, we have a result,

Theorem 2.3. Assume the same assumptions in Theorem 2.2. Then, for every δ_1

$$E_{\delta_1}[\phi_{1n}(F_{1n})] - E_{\delta_1}^*[\phi_{1n}(F_{1n}^*)] \to 0 \qquad (n \to \infty) .$$

Here $\phi_{1n}(F_{1n}^*)$ is an UMP invariant as seen in Noda and Ono (2000). Similarly to $\phi_{1n}(F_{1n})$, we can obtain an asymptotic validity of tests for linear hypotheses about $\underline{\tau}$ as well.

REFERENCES

Arnold, S. F. (1981). The Thory of Linear Models and Multivariate Analysis,John Wiley.

Noda, K. and Ono, H. (2000). UMP invariant tests in L. Model, Submitted to Commun. Statist.

RÉSUMÉ

La validité asymptotique de tests sur deux sortes d'hypothèses linéaires dans un modèle linéaire général est prouvée sous une structure d'une matrice covariance inconnue. Ce résultat obtenu est illustré par des exemples (qui seront donnés dans la présentation).