Developing New Theoretical Tools in Statistics Education Research

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1. Meaning of Statistical Objects and its Components

Recently some theoretical models have been proposed to describe statistical thinking by Wild & Pfannkuch (1999) and statistical literacy by Gal (in press). While these models can be useful at a macro-level of analysis to help curricula designers to take decisions about the "big" content areas that should be taught, statistics education is also requiring more specific micro-level models to analyse the students' statistical activity and to guide a systematic research programme in this field. In this paper we describe a theoretical model (Godino & Batanero, 1994; 1998), which has been successfully applied in different research work in statistics education and suggest a research agenda for statistics education based on the same. We will use the arithmetic mean as an example, although the theory is also valid for other types of statistical objects, such as theorems (e.g., central limit theorem) or a complete part of mathematics /statistics (e.g. variance analysis). In this model we distinguish five interrelated components in the meaning of the concept:

(1) *The field of problems from which the concept has emerged (phenomenological elements)*: One such problem in the case of the "mean" is finding the best estimation of an unknown quantity X when several different measurements $x_1, x_2,... x_n$, of the quantity are available. Other different situations are looking for an element $\bar{x}$, which is *representative* for a set of given values, symmetrically distributed, finding a fair amount to be shared out in order to achieve a uniform distribution or finding the most probable value for a random variable (expected value).

(2) *The representations of the concepts (representational elements)*; to solve the problems to refer to the problem data or its solution we need ostensive representations, such as the words "mean", "average", "expected value", the symbol $\bar{x}$, or graphical representations.

(3) *The procedures and algorithms (procedural elements)* to deal with a concept, to solve related problems or to compute its value in a given context, such as adding the quantities $x_1, x_2,... x_n$, and dividing by the number of data or computing a weighted average.

(4) *The definitions of the concept, its properties and relationships* to other concepts (*conceptual or intensive elements)*, such as the fact that the mean of a set of integer data can be a non integer number or that can be influenced by extreme values.

(5) *The arguments and proofs (validative elements)* we use to convince others of the validity of our solutions to the problems or the truth of the properties related to the concepts.

When teaching any other statistics concept, these five different types of knowledge should be considered and interrelated. Consequently, understanding the mean is a continuous constructive process where students progressively acquire and relate the different elements of the meaning of the concept.

2. Semiotic Functions. Elementary and Systemic Meanings. Reasoning as a Chain of Semiotic Functions

To describe statistical reasoning it is useful to consider semiotic functions: "*there is a semiotic function when an expression and a content are put in correspondence*" (Eco, 1979, p.83). The original in this correspondence is the significant, the image is the *meaning*. An *elementary meaning* is produced with a semiotic act in which a person relates an expression to a specific content. For example the symbol $\overline{x}$ can represent the average algorithm; the expression 'mathematical expectation', can refer to abstract concepts. Semiotic processes can also produce *systemic meanings*. For example when we speak of studying the "mean", we refer to the whole system of practices associated with the mean. When carrying out any statistical activity or reasoning, one or more semiotic functions appear among the entities described in section 2 and statistical reasoning can be described by a sequence of semiotic functions. Below we analyse, as an example, the response by a grade 10 student to a task given by Watson and Moritz (2000) a part of her study on averages:

Task: Let's say that the average for 10 families is 2.3 children. If the Grants have 4 children and the Coopers have 1 child, show how many children the other 8 families might have (Watson & Moritz, 2000, p. 19).

Response: They might have two or three because you add it all up. Say another four families have two, and another three, oh , ...another four families have three. Four times 3 is 12 and 2 times 4 is 8. Eight plus 12 plus 5 should be 25. Ten goes into 25, 2.3 or something like that. (p. 26).

Analysis

(1)*They might have two or three:* The child refers to a property he attributes to the average that the data values should be close to the average. He is implicitly establishing a correspondence between the word "average" in the task statement and the idea of mode. With the words "two", "three" he refers to integer numbers and also to imaginary families with that number of children.

(1) *because you add it all up:* There is an implicit reference to the arithmetic mean; where all the data are added up to get the average value. The child uses this sentence as an explanation of the fact that the particular value of the average in the problem (2.3).

(2) *Say another four families have two, and another three, oh , ...another four families have three:* The child is not able to invert the mean algorithm and, try to solve a problem by trial and error; he imagines a possible distribution of the number of children in the eight remaining families; he assigns two children to half the families and three to the other half; he uses implicitly the ideas of mode and number, and refer to particular types of families (families with 2, 3 children).

(3) *Four times 3 is 12 and 2 times 4 is 8:* He is describing a series of imagined multiplications. The idea of multiplication and their results are also referred.

(4) *Eight plus 12 plus 5 should be 25:* He describes an action (adding three numbers), refers to

addition and its results.

(5) *Ten goes into 25, 2.3 or something like that:* Here the ideas of mean as an operation and its results is evoked. Since the result is not what is expected (2.3), the child supposes the result can be approximated.

This example shows the complexity of solving even elementary statistical problems. Very often students are not able to establish the correct semiotic functions and this can explain their errors and difficulties. In the example above, the children mix the ideas of mode and mean and, although he can perform the mean algorithm, is not able to invert the algorithm to solve the problem.

3. Institutional and Personal Meanings. Assessing Knowledge

Different levels of abstraction can be considered for each of the five components defined above, and thus, the *meaning of the mean* is very different at different institutions. Primary school children can give a simple definition of the mean, and recognise a simple notation. A statistical literate citizen also understands the use of means in the mass media or in the business world. In scientific or professional work, or at university level, however, mean is given a more complex meaning. We also distinguish between the *personal* and *institutional* meaning to differentiate between the meaning that for a given concept has been fixed in a specific institution, and the meaning given to the concept by a particular person in the institution. For example, a primary school child might be surprised when obtaining a non integer value (e.g.2.3) for the mean number of children in a family and interpret this mean that *"the average is of the older children, which they could say are fully grown... and the .3 is a child that is growing up to be an older child. So that, like, say the kid is 3 now, once it turns to be 10, it will get to be 1, so they will have three children* (Watson & Moritz, 2000, pp. 35-36)". The aim of teaching statistics is to help students to progressively match their personal meanings to the meaning that statistics concepts receive in a teaching institution.

A main problem in didactic research is assessment. Since knowledge is an unobservable construct a students' understanding about a specific object should be deduced from his practices (arguments, procedures, representations, properties he/ she assign to the object) in specific assessment tasks. Therefore, problems of validity and reliability arise since there is always a sampling from all the possible tasks we could give the students as regards the object and another sampling from all the possible responses the same student could give to the same assessment task. Once this complexity is recognised, a phenomenological and epistemological study will serve to determine the institutional meaning for an object and to provide criteria to design relevant assessment situations.

4. A Research Agenda

To finish we show the utility of this model in setting a basic research agenda in Statistics Education. We classify the research questions according to two different dimensions (Table 1):

(1) The aim of the research where we distinguish three different categories: The characterisation of institutional and personal meanings - or *semiometry* -, the study of factors that affect meaning

and the study of interdependence of meanings - *ecology*- and the study of change of meaning over time - *dynamics*.

(2) The research focus: a researcher might be interested in the institutional meaning, the personal meaning or the interaction between them (instruction).

Table 1. Classification of research problems as regards the aim and focus

Research Focus	Research Aim SEMIOMETRY Measuring / Describing	 ECOLOGY Finding Relationships	 DINAMICS Studying Changes
EPISTEMOLOGY (Institutional meanings)	What is the institutional meaning of O?	What are the relationships of O with other Objects? What factors affect institutional meaning?	How the institutional meaning of O changes in a given time?
COGNITION (Personal meanings)	What is the personal meaning of O? What personal meaning is applied during a problem solving process ?	What relationships does the person establish between O and other objects? What factors affect personal meaning?	How the personal meaning of the object changes in time or as a consequence of instruction?
INSTRUCTION (Interaction between Institutional and personal meanings)	How is instruction on O organised?	How different factors affect instruction? How to design instruction taking these factors into account?	How instruction develops through time?

Examples of statistical education research that can classified in each cell will be given in a wider version of this paper will be located at http://www. ugr.es/local/batanero/. We are conscious of the difficulty of theoretical research, and of the fact that statistical knowledge, reasoning, teaching and learning are very complex to be described in just one model. However, we are also convinced of the necessity and utility of carrying out theoretical reflections if we want statistics education to develop towards an autonomous research discipline.

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RESUME

Nous présentons un modèle théorique du signifié des objets mathématiques que nous avons utilisé dans plusieurs thèses réalisées à l'Université de Grenade. Nous en déduisons une agenda pour la recherche en didactique de la statistique.