An Illustration of the Causality Relation between Government Spending and Revenue Using Wavelets Analysis on Finish Data.

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ABSTRACT

Quarterly data for the period 1960:1 to 1997:2, conventional tests, a bootstrap simulation approach and a multivariate Rao's *F*-test have been used to investigate if, the causality between government spending and revenue in Finland have been changed at the beginning of 1990 due to future plans to create the European Monetary Union (EMU). The results indicated that during the period before 1990, the government revenue Granger caused spending, while the opposite has happened after 1990, which agrees better with Barro's tax smoothing hypothesis. However, when using monthly data instead of quarterly data for almost the same sample period, totally different results have been noted.

The general conclusion is that the relationship between spending and revenue in Finland is still not completely understood. The ambiguity of these results may well be due to the fact that there are several time scales involved in the relationship, and that the conventional analyses may be inadequate to separate out the time scale structured relationships between these variables. Therefore, to empirically investigate the relation between these variables we attempt to use the *wavelets* analysis that enables to separate out different time scales of variation in the data. We find that time scale decomposition is very important for analysing these economic variables.

Key words : Wavelets; Timescale; Causality tests; Spending; Revenue; EMU

JEL Classification: C32, H62

1. INTRODUCTION

Wavelet is a fairly new approach in analysing data (e.g. Daubechies 1992) that is becoming increasingly popular for a wide range of applications (e.g. statistics, time series analyses). This subject is not really familiar in econometrics, however, and very few studies have used the wavelets in econometric applications (e.g. Ramsey and Lampart 1998, Goffe, W.L. 1993).

Ramsey and Lampart (1998) have used the wavelet analysis and found it useful in studying the relationship between money and income. The idea was based on the fact that the time period (time scale) of the analysis is very crucial for determining those aspects of decision making that are relatively more important, and those that are relatively less important. Moreover, they stated that in econometrics one can envisage a cascade of time scales within which different levels of decisions are being made. Some decisions are taken with long horizons, others with short horizons. The authors used the US Federal Reserve Board as an example to show that the choice of the time scale determines not only the length of the period over which one requires forecasts of future events, but also the very choice of the variables that are to be the focus of the decision maker's processing of information. They empirically investigated the relation between money and income by using wavelet analysis that enables to separate out different time scales of variation in the data. Shortly speaking, they investigated the role of time scale in economic relationships in terms of money and income relationship.

In this paper we use the wavelets analysis in studying the relationship between government spending and revenue in Finland. There is some controversy about the nature of the relationship between spending and revenue and the extent to which the relevant theory is supported by the empirical evidence.

The issue of curtailing budget deficits is one of the central themes of economic policy in many member countries of the European Union (EU) and is one of the key convergence criteria of the European Monetary Union (EMU) membership. Correcting fiscal imbalances is a necessary precondition for the EMU membership. As a matter of fact, government spending has often exceeded government revenues in almost all member counties of the EU.

The question of interest focuses on the causal nexus of government spending and revenue in the new member countries of the EU. Hence, it is important to investigate whether the political system first decides how much to spend and then decides how much to bring in as revenue. In other words, we are investigating whether the decisions regarding the amount of spending in these countries precede the decisions regarding the amount of taxes, or if connection is the other way around, or if these decisions are taken simultaneously.

Another question of interest is how the convergence criteria of the EMU affect the causality nexus? That is, has the causality changed at the beginning of 1990 because of the future plans of creating the EMU.

Shukur and Hatemi-J (1998), and Hatemi-J and Shukur (1999), investigated this subject and tried to answer analytically these questions regarding government financial policy in Finland. In Shukur and Hatemi-J (1998), the authors applied a VAR model and a VECM on quarterly data and found that only government revenue Granger causes spending for the sample period 1960:1 to 1997:2. This result did not accord with Barro's (1979) tax smoothing hypothesis, which assumes that causality runs from government spending to revenue. This hypothesis takes the path of government spending to be exogenous and taxes are adjusted to minimise distortion, while the budget is balanced intertemporarilly. However, in order to answer the question whether the causality changed at the beginning of 1990 because of the future plans of creating EMU, the authors split the sample into two subsamples, before and after 1990, and separately performed tests for Granger causality between spending and revenue in each subsample. They found that the causality nexus proved to exist from spending to revenue in the last subperiod, which agrees better with Barro's tax smoothing hypothesis. That is, the causality has changed direction at the beginning of 1990. One can, of course, think about this result as if the change might have happened due to the future plans of creating the EMU.

In Hatemi-J and Shukur (1999) the authors used different test methods for the same purpose. In addition to the single equation Likelihood Ratio (LR) test for causality they used two other tests, the systemwise Rao's *F*-test (Rao, 1973), developed by Shukur and Mantalos (2000), and the bootstrap test developed by Mantalos (1998). The results from this study have been shown to be similar to those found by Shukur and Hatemi-J (1998). The Rao's *F*-test has been found to work very well in integrated cointegrated VAR systems, while the bootstrap test proved to work well even in such situations where the systems are not cointegrated. Note that in the Shukur and Mantalos test (2000) the authors use the Ordinary Least Squares (OLS) method, while we in this study use Zellner's Iterative Seemingly Unrelated Regression (ISUR) method. The ISUR technique provides parameter estimates that converge to the maximum likelihood parameter estimates which are unique.

In this paper, however, when using monthly data instead of quarterly data for almost the same sample period different results have been noted. The results obtained by using the monthly data have shown that there exist feedback relations (i.e. in two directions) between these variables over the entire sample period, 1960:01 to 1998:09. When splitting the data into two subsamples, 1960:01 to 1989:12 and 1990:01 to 1998:09, similar results have been noted. These results are obtained by applying the three different test methods, i.e. the LR test, systemwise Rao's F-test and the bootstrap test.

Therefore, the general conclusion is that the relationship between spending and revenue in Finland is still not understood completely. The ambiguity of these results may well be due to the fact that there are several time scales involved in the relationship, and that the conventional analysis may be inadequate to separate out the time scale structured relationships between the variables.

Here we attempt to shed light on this issue by separating the empirical analysis of the relationship between the variables into that between the variables after separation into time scale components. Instead of considering the net relationship over all time scales as in the conventional analysis, we (as in Ramsey and Lampart, 1998) consider a set of relationships, one for each time scale.

The paper is organised as follows: Section 1 gives an introduction. In Section 2 we introduce the wavelets analysis, while in Section 3 we present the data used in this study. In Section 4 we present the model and the methodology. Section 5 describes estimation and testing procedures, and Section 6 presents the estimated results. Finally, we give a short summary and conclusions in Section 7.

2. WAVELET ANALYSIS

The wavelet transform has been expressed by Daubechies (1992) as "a tool that cuts up data or functions into different frequency components, and then studies each component with a resolution matched to its scale". Thus, with wavelet transform one can analyse series with heterogeneous (unlike Fourier transform) or homogeneous information at each scale. Unlike the Fourier transform, which uses only sins and cosines as basis functions, the wavelet transform can use a variety of basis functions.

The wavelet decomposition in this paper is made with respect to the so called Symlets basis, and we will hereby give a brief presentation about this decomposition methodology. Let $\mathbf{X} = (X_0, X_2, ..., X_{T-1})'$ be a column vector containing T observations of a real-valued time series, and assume that T is an integer multiple of 2^M, where M is a positive integer. The discrete wavelet transform of level J is an orthonormal transform of $\mathbf{X}$ defined by

$$\mathbf{d} = (\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_j, \ldots, \mathbf{d}_J, \mathbf{s}_J)' = \mathbf{WX},$$

where $\mathbf{W}$ is an orthonormal $T \times T$ real-valued matrix, i.e. $\mathbf{W}^{-1} = \mathbf{W}'$ so $\mathbf{W}'\mathbf{W} = \mathbf{WW}' = \mathbf{I}_T$. $\mathbf{d}_j = \{d_{j,k}\}$, $j = 1,2,\ldots, J$, are $T/2^j$ ´1 real-valued vectors of wavelet coefficients at scale j and location k.

The real-valued vector $\mathbf{s}_J$ is made up of $T/2^J$ scaling coefficients. Thus, the first T - $T/2^J$ elements of $\mathbf{d}$ are wavelet coefficients and the last $T/2^J$ elements are scaling coefficients, where $J \leq M$. Notice that the length of $\mathbf{X}$ does coincide with the length of $\mathbf{d}$ (length of $\mathbf{d}_j$ = 2^{M-j}, and $\mathbf{s}_J$ = 2^{M-J}).

In practice, the discrete wavelet transform is applied without exhibiting the matrix $\mathbf{W}$, and we therefore use a fast filtering algorithms of order $O(n)$ based on so called quadrature mirror filters that uniquely correspond to the wavelet of interest, see Mallat (1989). In what follows, we will merely consider the wavelet in terms of filters.

Now, let $\{h_1\} \equiv \{h_{1,0}, \ldots, h_{1,L-1}\}$ denote the wavelet filter coefficients of a Daubechies compactly supported wavelet of width L, where $L < T$, and let $\{g_1\} \equiv \{g_{1,0}, \ldots, g_{1,L-1}\}$ be the corresponding scaling filter coefficients, defined via the quadrature mirror relationship,

$$h_l = (-1)^l g_{L-1-l} \qquad \text{for} \qquad l = 0, \ldots, L-1.$$

It is important to note that the first kind of wavelet filter is called the Haar wavelet, (Haar 1910), which is a filter of width $L = 2$, that can be defined either by its wavelet coefficients,

$$h_0 = \frac{1}{\sqrt{2}}, \text{ and } h_1 = -\frac{1}{\sqrt{2}},$$

or, equivalently, by its scaling coefficients,

$$g_0 = g_1 = \frac{1}{\sqrt{2}}.$$

The Haar wavelet is special since it is the only compactly supported (zero outside a finite interval) orthogonal wavelet that is symmetric.

However, Daubechies (1992) developed a finite number of filter coefficients that are not only orthonormal, but also have compact support, i.e. the Daublets 'D(L)' and the Symmlets 'S(L)'. Note that the Daublets are quite asymmetric while Symmlets were constructed to be as nearly symmetric as possible.

The filters can be applied to any sequence $a = \{a_T\}$ through the operators $\mathbf{H}$ and $\mathbf{G}$

$$(\mathbf{H}a)_k = \sum_T h_{T-2k} a_T \; ; \qquad (\mathbf{G}a)_k = \sum_T g_{T-2k} a_T \; .$$

An application of operator $\mathbf{H}$ and $\mathbf{G}$ corresponds to one step in the discrete wavelet transformation. The complete discrete wavelet transformation is a process that recursively applies to the above equation.

The algorithm starts by applying the filters to the data vector $\mathbf{X}$ and obtains the sub-vector of wavelet coefficients $\mathbf{d}_1 = \mathbf{HX}$ together with the corresponding smooth coefficients $\mathbf{s}_1 = \mathbf{GX}$ at level $j = 1$. The procedure continues by applying the operators again to $\mathbf{s}_1$ to obtain $\mathbf{d}_2 = \mathbf{Hs}_1 = \mathbf{GHX}$ and $\mathbf{s}_2 = \mathbf{G}^2\mathbf{X}$, and so on until reaching the last scale J. The wavelet decomposition of the vector $\mathbf{X}$ can be represented as the vector $\mathbf{d}$ of the same size, given by

$$\mathbf{d} = (\mathbf{HX}, \mathbf{GHX}, \mathbf{GH}^2\mathbf{X}, ..., \mathbf{GH}^{J-1}\mathbf{X}, \mathbf{G}^J\mathbf{X})' \; .$$

For more details about the wavelet transform in terms of operators, see Strang and Nguyen (1996). The multiresolution analysis of the data leads to a better understanding of wavelets. The idea behind multiresolution analysis is to express $\mathbf{W}'\mathbf{d}$ as the sum of several new series, each of which is related to variations in $\mathbf{X}$ at a certain scale.

Now, since the matrix $\mathbf{W}$ is orthonormal we can reconstruct our time series from the wavelet coefficients $\mathbf{d}$ by using

$$\mathbf{X} = \mathbf{W}'\mathbf{d} \, .$$

We partition the columns of $\mathbf{W}'$ commensurate with the partitioning of $\mathbf{d}$ to obtain

$$\mathbf{W}' = [\mathbf{W}_1 \; \mathbf{W}_2 \ldots \mathbf{W}_J \; \mathbf{V}_J],$$

where $\mathbf{W}_j$ is a $T \times T/2^j$ matrix and $\mathbf{V}_J$ is a $T \times T/2^J$ matrix. Thus, we can define the multiresolution analysis of a series by expressing $\mathbf{W}' \, \mathbf{d}$ as a sum of several new series, each of which is related to variations in $\mathbf{X}$ at a certain scale:

$$\mathbf{X} = \mathbf{W}' \, \mathbf{d} = \sum_{j=1}^{J} \mathbf{W}_j \, \mathbf{d}_j + \mathbf{V}_J \, \mathbf{s}_J = \sum_{j=1}^{J} \mathbf{D}_j + \mathbf{S}_J \; .$$

The terms in the previous equation constitute a decomposition of $\mathbf{X}$ into orthogonal series components $\mathbf{D}_j$ (detail) and $\mathbf{S}_J$ (smooth) at different scales, and the length of $\mathbf{D}_j$ and $\mathbf{S}_J$ coincides with the length of $\mathbf{X}$ ($T \times 1$ vector). Because the terms at different scales represent components of

X at different resolutions, the approximation is called a multiresolution decomposition, see Percival and Mofjeld (1997).

As we mentioned earlier the wavelet decompositions in this paper will be made with respect to the Symlets basis. This has been done by using the S-plus Wavelets package produced by StatSci of MathSoft that was written by Bruce and Gao (1996). Figure 2 and Figure 3 show the multiresolution analysis of order $J = 6$ based on S(8) wavelet filter.

When choosing a specific kind of wavelets several factors should be taken into consideration. Two such important factors are the smoothness and the spatial localisation of the wavelet. In general the wavelets with a wider support (L is big) are smoother but spatially less localised, while the wavelets with a narrow support (L is small) are more spatially localised but less smooth. To get a reasonable degree of smoothness without loosing the property of spatial localisation we use quite a moderate size of L, i.e. $L = 8$. The Wavelet filter coefficients for the Symmlets of length 8, i.e. S(8), are given as follows:

$h_0 = 0.07576571$, $h_1 = -0.02963553$, $h_2 = -0.4976187$, $h_3 = 0.8037388$,
$h_4 = -0.2978578$, $h_5 = -0.09921954$, $h_6 = 0.01260397$, $h_7 = 0.0322231$.

Recall that the scaling filter is related to the wavelet filter via the quadrature mirror filter relationship given by equation (1).

Note that Ramsey and Lampart (1998) have used $L = 10$. Here, to investigate whether the use of other sizes of L has any impact on the results of the study, $L = 6$ and $L = 10$ have been used in some experiments, but we did not find any noticeable effects on our inferential statements.

3. DATA

The investigation of the causal relationship between government spending (S) and government revenue (R) is performed by using monthly data that are drawn from the *International Monetary Found* (IMF), and cover the period 1960:01 through 1998:9. The variables are chosen to be in logarithmic form, and hereafter will be referred to as *lnS* and *lnR*, respectively.

As mentioned in Section 2 wavelet can help us in decomposing the original series into a set of orthogonal series components that provide representations of the original series. Usually a series is decomposed into six different components, e.g. D1, D2, . . ., D6, that stand for different frequency intensities in the original series, and a last component (S6) which stands for the long run trend in the series. To explain, the time scale D1 stands for the finest level in the series and represents the highest frequency that occurs at the one-month scale. In the same manner, the D2 can stand for the next finest level in the series and represents the two-month scale, D3 for the four-month-scale, D4 for the eight-month scale, D5 for the sixteen-month scale and finally, D6, which may stand for the 32-month scale.

In addition to the monthly data we examined the relationship between the revenue and spending when the variation in each variable has been restricted to a specific scale, i.e., when the variables are transformed by wavelet transformation into the different time scales, D1 to D5 (see Figures 1 and 2). We did not use the D6 scale since it was difficult for us to find a useful interpretation of a 32-month scale. Note that the S6, in these figures, stands for the log term trend and has not been considered in our causality analysis.

Using wavelet, we reconstructed the above mentioned time series by time scales and the relations between the variables at each time scale. We then examined and illustrated how the extent to which an allowance for different effects by scale and variations in the relationships over time leads to insight into the total variation of the signal over time.

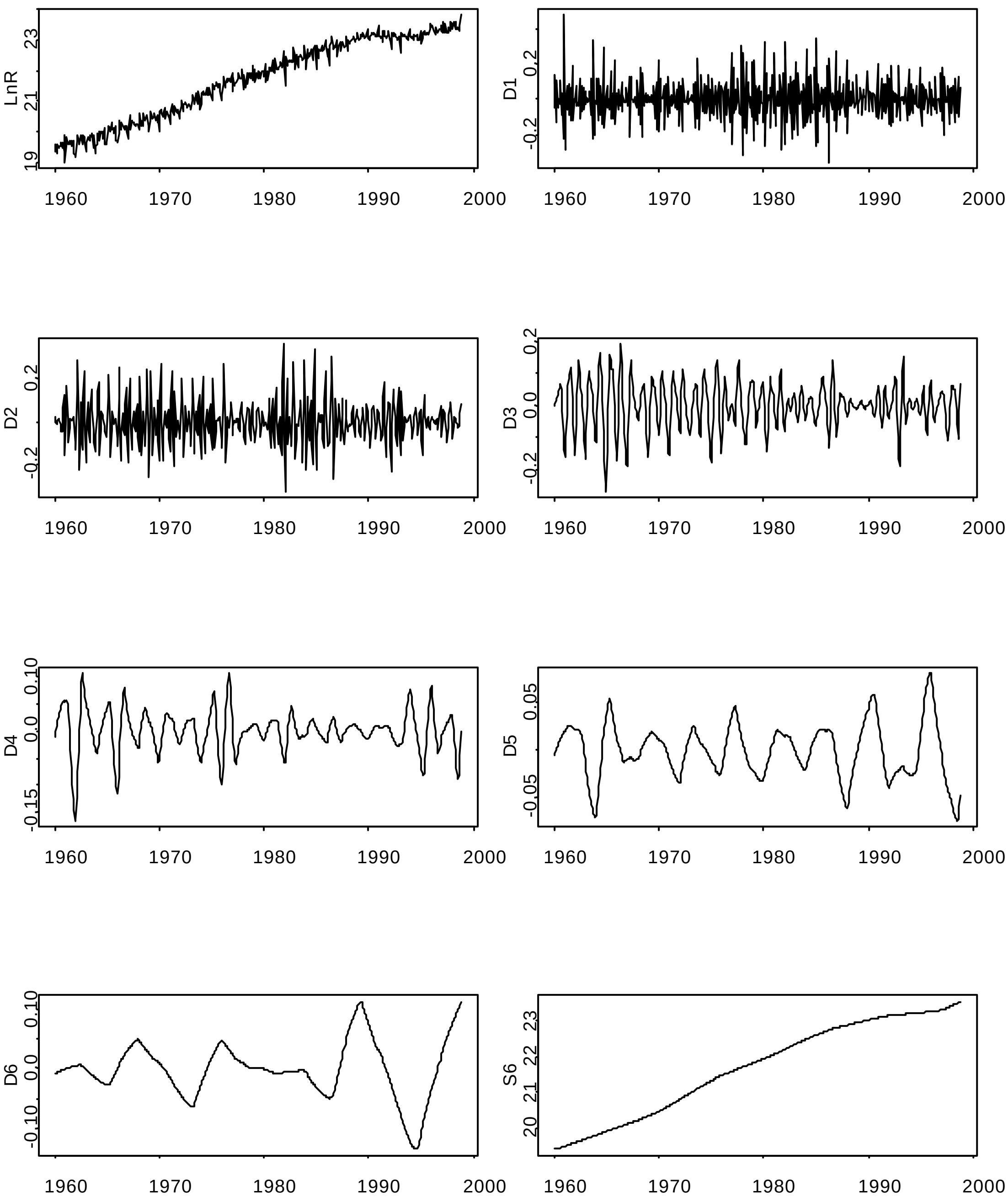


Figure 1. Time series plots of data for *LnR* and their different scales.

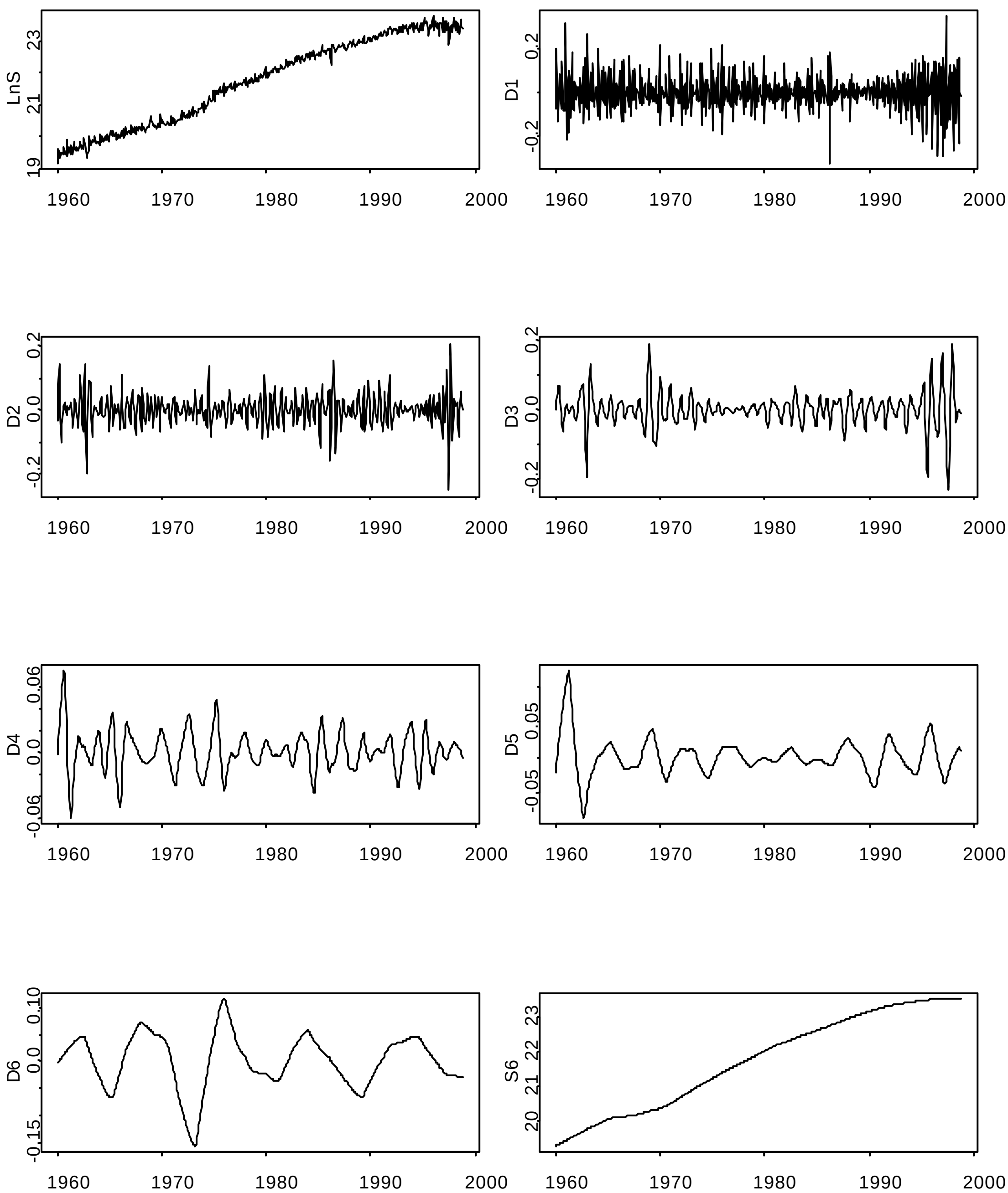


Figure 2. Time series plots of data for *LnS* and their different scales.

4. MODEL SPECIFICATION AND TESTING METHODOLOGY

By causality we mean causality in the Granger (1969) sense. That is, we would like to know if one variable precedes the other variable or if they are contemporaneous. The Granger approach to the question whether *lnS* causes *lnR* is to see how much of the current value of the second variables can be explained by past values of the first variable. *lnR* is said to be Granger-caused by *lnS* if *lnS* helps in the prediction of *lnR*, or equivalently, if the coefficients of the lagged *lnS* are statistically significant in a regression of *lnR* on *lnS*. Empirically, one can test for causality in Granger sense by means of the following vector autoregressive (VAR) model:

$$\ln R_t = a_0 + \sum_{i=1}^{k} a_i \ln R_{t-i} + \sum_{i=1}^{k} b_i \ln S_{t-i} + e_{1t} \ , \qquad (1)$$

$$\ln S_t = c_0 + \sum_{i=1}^{k} c_i \ln R_{t-i} + \sum_{i=1}^{k} f_i \ln S_{t-i} + e_{2t} \ , \qquad (2)$$

where e_{1t} and e_{2t} are error terms, which are assumed to be independent white noise with zero mean. The number of lags, k, will be decided by using the Schwarz (1978) information criteria, the Hanna and Quinn (1971) criteria and the systemwise likelihood ratio (LR) test. In order to see if the variables are cointegrated (i.e. if there exists any long run relationship between the variables) we first test for integration of each variable. A variable is integrated of order d, denoted I(d), if it must be differenced d times to achieve stationarity. We use the augmented Dickey-Fuller (1979, 1981), in what follows referred to as ADF, tests for deciding the integration order of each aggregate variable. The distinction between stationary I (0) and non-stationary I (1) processes is a first step in analysis of time series. Several authors take the first difference to remove nonstationarity, while others are restrictive against differencing believing that information will be lost. Note that, to achieve stationarity, Ramsey and Lampart 1998 have used the data in logarithmic differenced form. Hence, to avoid any eventual loss of information, we in this study use the original series but in logarithmic form.

In the rest of this section we will present the different approaches tests for causality that we use in this study, i.e. the conventional singlewise (LR) test, and the two recommended, Rao's *F*-test and the Bootstrap test mentioned in Shukur and Mantalos (2000) and Mantalos (1998), respectively.

4.1. Conventional Causality Test (singlewise LR test)

According to Granger and Newbold (1986) we can test for causality in the following way:
We construct a joint *F*-tests for the inclusion of lagged values of *lnS* in (1) and for the lagged values of *lnR* in (2). The null hypothesis for each *F*-test is that the added coefficients are zero and therefore

the lagged *lnS* does not reduce the variance of *lnR* forecasts (i.e. b_i in (1) are jointly zero for all *i*), or that lagged *lnR* does not reduce the variance of *lnS* forecasts (i.e. f_i in (2) are jointly zero for all *i*). If neither null hypothesis is rejected, the results are considered as inconclusive. On the other hand, if both of the *F*-tests rejected the null hypothesis, the result is labelled as a feedback mechanism. A unique direction of causality can only be indicated when one of the pair of *F*-tests rejects and the other accepts the null hypothesis.

4.2. The Systemwise Rao's *F*-test

In this subsection we present the Granger-causality test by using the multivariate Rao's *F*-test. Consider the following VAR(p) process:

$$\mathrm{y}_t = \eta + \mathrm{A}_1 \mathrm{y}_{t-1} + \ldots + \mathrm{A}_p \mathrm{y}_{t-p} + \varepsilon_t, \qquad (3)$$

where $\boldsymbol{e}_t = (\boldsymbol{e}_{1t}, \ldots, \boldsymbol{e}_{kt})'$ is a zero mean independent white noise process with nonsigular covariance matrix Σ_e and, for j = 1, ... , k, $\mathrm{E}|\boldsymbol{e}_{jt}|^{2+t} < \infty$ for some > 0. The order p of the process is assumed to be known. Now, by portioned y_t in (m) and (k-m) dimensional sub-vectors y_t^1 and y_t^2 and A_i matrices portioned comformably then y_t^2 does not Granger-cause the y_t^1 if the following hypothesis:

$$H_0 = \mathrm{A}_{12,i} = 0 \quad \text{for} \quad i = 1, \Lambda \ , p\text{-}1 . \qquad (4)$$

is true.

Let us define:

$\mathrm{Y}: = (\mathrm{y}_1, \Lambda\ , \mathrm{y}_T)$ $(\mathrm{k} \times \mathrm{T})$ matrix,

$\mathrm{B}: = (\mathrm{v}, \mathrm{A}_1, \Lambda\ , \mathrm{A}_p)$ $(\mathrm{k} \times (\mathrm{kp} + 1))$ matrix,

$$\mathrm{Z}_t: = \begin{bmatrix} 1 \\ \mathrm{y}_t \\ \mathrm{M} \\ \mathrm{y}_{t-p+1} \end{bmatrix} \quad ((\mathrm{kp} + 1) \times 1) \text{ matrix,}$$

$\mathrm{Z}: = (\mathrm{Z}_0, \Lambda\ , \mathrm{Z}_{T-1})$ $((\mathrm{kp}+1) \times \mathrm{T})$ matrix, and

$\delta: = (\varepsilon_1, \Lambda\ , \varepsilon_T)$ $(\mathrm{k} \times \mathrm{T})$ matrix.

By using these notations, for t = 1, .,.,T, the VAR (p) model including a constant term (ν) can be written compactly as:

$$Y = BZ + \delta. \qquad (5)$$

We first estimate model (5), equation by equation, using the OLS method. The whole VAR system is then estimated using Zellner's Iterative Seemingly Unrelated Regression (ISUR) method. As we previously mentioned, the ISUR technique provides parameter estimates that converge to the maximum likelihood parameter estimates which are unique.

Let us denote by $\hat{\delta}_U$, the ($k \times T$) matrix of estimated residuals from the *unrestricted* regression (3), and by $\hat{\delta}_R$ the equivalent matrix of residuals from the *restricted* regression with H_0' imposed. The matrix of cross-products of these residuals will be defined as $S_U = \hat{\delta}_U'\hat{\delta}_U$ and $S_R = \hat{\delta}_R'\hat{\delta}_R$ respectively. The Rao's F-test can be then written as:

$$RAO = (f/q)(U^{1/s} - 1) \qquad (6)$$

where $s = \sqrt{\dfrac{q^2 - 4}{k^2(G^2 + 1) - 5}}$, $\phi = \Delta s - r$, $\Delta = T - (k(kp+1) - Gm) + ½[k(G-1) - 1]$, $r = q/2 - 1$, and $U = \det \mathbf{S}_R / \det \mathbf{S}_U$. $q = Gm^2$ is the number of restrictions imposed by H_0, where G is the p restriction in (3) and m is the dimension of the sub-vector y_t^1. RAO is approximately distributed as $F(q,\ f)$ under the null hypothesis, and reduces to the standard F statistic when $k = 1$.

4.3. The Bootstrap Testing Approach

In this subsection we present the Bootstrap testing procedure (Efron, 1979). Generally, the distributions of the test statistics we use are known only asymptotically, which means that the tests may not have the correct size, and inferential comparisons and judgements based on them could be misleading. However, several studies (e.g. Horowitz, 1994; Mantalos and Shukur, 1998; and Shukur and Mantalos, 1997), have shown the robustness of the bootstrap critical values.

From regression (5), a direct residual resampling gives:

$$Y^* = \hat{B}Z^* + \delta^* \qquad (5a)$$

where δ^* are i.i.d. observations $\delta_1^*, ..., \delta_T^*$, drawn from the empirical distributions ($\hat{F}_\delta$) putting mass 1/T to the adjusted OLS residuals ($\hat{\delta}_i - \overline{\delta}$), i = 1, . . ., T. The basic principle of the Bootstrap testing is to draw a number of Bootstrap samples from the model under the null hypothesis, calculate the Bootstrap test statistic (T_s^*). The Bootstrap test statistic (T_s^*) can then be calculated by repeating this step N_b number of times. We then take the (α):*th* quantile of the bootstrap distribution of T_s^* and obtain the α-level "bootstrap critical values" (c_{ta}^*). We then calculate the test statistic (T_s) which is the estimated test statistic. Finally, we reject the null hypothesis if $T_s \leq c_{ta}^*$.

As regards N_b, the number of the bootstrap samples used to estimate bootstrap critical value, Horowitz (1994) used the value of N_b=100. In this study we follow the recommendation in Davidson and Mackinnon (1996) and use N_b=1000 to estimate the P-value.

5. ESTIMATION AND TESTING PROCEDURES

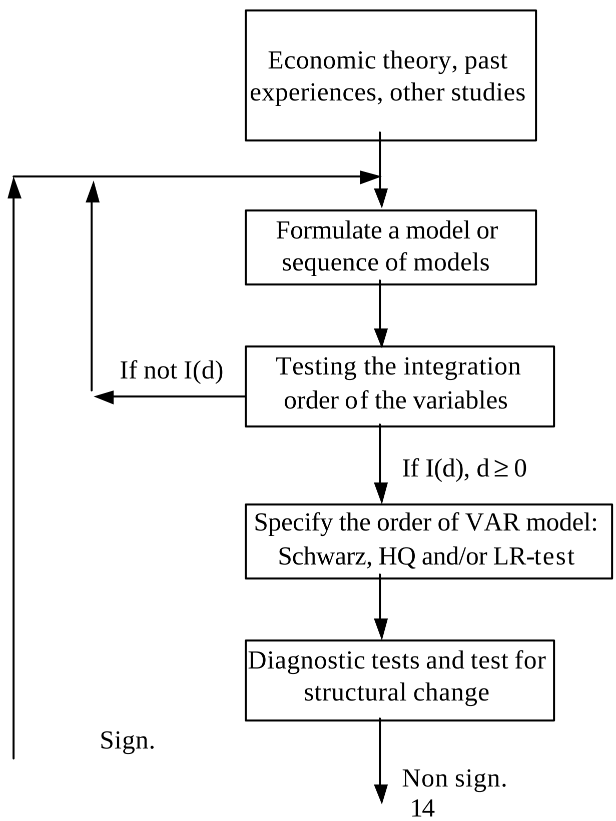
In this paper, we intend to study the causal nexus of government spending and revenue in Finland by constructing a vector autoregressive (VAR) model that allows for causality test in the Granger sense. For this purpose, we propose a simple strategy for how to select an appropriate model by successively examining the adequacy of a properly chosen sequence of models, using both single equation and systemwise tests. Note that the methodology used for misspecification testing in this paper follows the ideas described in Godfrey (1988). We apply his line of reasoning to the problem of autocorrelation, and then extend it to other forms of misspecification. Systemwise Rao's *F*-test have been adopted to test the adequacy of the model. If the systemwise misspecification tests lead to rejection, single equations tests will be conducted to identify specific equation/s that may lead to misspecification.

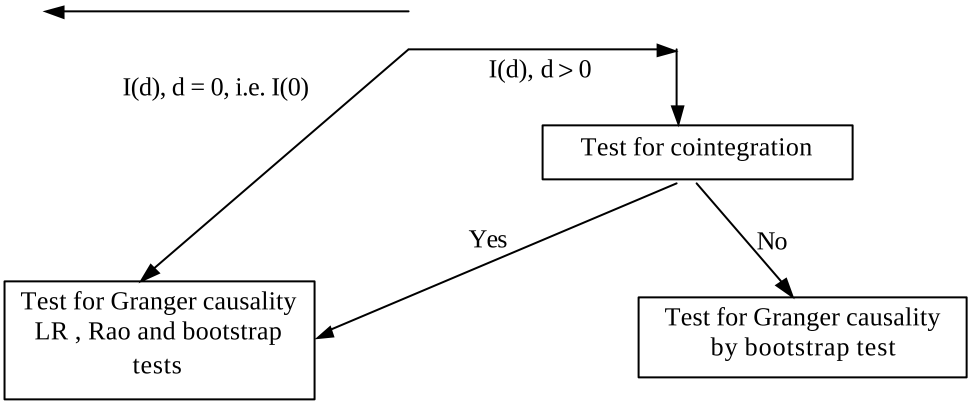
Our aim is to find a well-behaved model, which satisfies its underlying statistical assumptions, and which at the same time agrees with theoretical restrictions of economic theory. Given such a model, we then test for the presence and direction of the causality, and draw some conclusions about the study. In this study we use the standard program packages EViews, RATS, CATS, S-plus, and Gauss.

In Figure 3, we present an outline of our strategy for how to solve issues regarding specification of models and choice of proper ways to tackle situations that can arise with non-stationary series.

In order to construct a model to fit a specific data set, model builders make use of prior information derived from economic theory and previous experiences. In our study of the relationship between government spending and revenue in Finland we try to construct, estimate, test, and analyse a VAR model that adequately represents the relationship and mimic the true data-generating process. Given that such a model exists, we will then study other theoretical restrictions imposed by economic theory, and draw some important inferential statements.

Figure 3: Model Selection and Testing Strategy Outline





First, we use the ADF test to consider the integration nature of the variables included in the VAR model. If the variables are stationary, i.e. I(0), we then apply the VAR model and carry out our estimation procedure. If the variables are non-stationary, any regression between them may be spurious. Accordingly, a test for cointegration has to be performed. Note that, according to the Granger (1969) representation theorem, the variables that are cointegrated have an error correction model (ECM) representation, and vice versa. Hence, another possibility for estimation is also available, i.e. the VECM. In this study, however, we only concentrate on applying the VAR model.

Second, we determine the suitable degree of the VAR process by considering a number of VAR models. We begin by estimating these models equation by equation using the OLS method.[1] We will then specify the order of the VAR model. There are two common ways, to do that, either using the LR test, or using some model selection criteria (e.g., the Schwarz (1978), Hannan and Quinn (1979) information criteria, in what follows referred to as SC and HQC respectively). The model that minimises these criteria will be selected.

Third, we apply a battery of diagnostic tests to ensure the appropriateness of the model and if the statistical assumptions are indeed satisfied. Now, if a model is subjected to several specification tests, one or more of the test statistics may be so large (or the p-values very small) that the model is clearly unsatisfactory. At that point one either has to modify the model or search for an entirely new one. For example, if the residuals appear to be autocorrelated, then we can reestimate the model using lag lengths that yield serially uncorrelated residuals. Hence, we may face a sequence of candidate models that are almost free from autocorrelation. In this case we recommend starting

[1] Note that the OLS estimates are both consistent and asymptotically efficient and that the Seemingly Unrelated Regressions (SUR) do not add to the efficiency of the estimation procedure since each equation in the VAR model contains the same right-hand-side variables. The Iterative Seemingly Unrelated Regressions (ISUR) method should be used, however, if cross-equation restrictions are imposed. In such case the ISUR technique provides parameter estimates that converge to the maximum likelihood parameter estimates, which are unique.

from the beginning, using the models selection criteria, and to perform other diagnostic tests to choose the proper model.

Fourth, if the chosen model is shown to be fairly adequate, we reconsider the integration nature of the included variables and proceed as follows:

When the variables are stationary, we will continue our procedure by testing the variables included in the selected VAR model for causality, in the Granger sense, using any of the previously mentioned tests. Here, we can verify if there is any causal nexus between the variables. On the one hand, if the variables are not stationary, we test for cointegration between them using the Johansen (1988) procedure. If there is indication of cointegration, we can still use any of the tests to investigate the causality relationship between the variables. On the other hand, in the case of non-cointegration between the variables, then only the bootstrap test approach can support us with reliable results for causality. By following the strategy outlined here, one can avoid inadequate models that might lead to misleading results and inferences.

6. RESULTS

In this section we present the estimation results of applying the following testing and estimation procedure. Firstly, we test the variables (i.e. those of the monthly data and those that are produced by the wavelets transformation) for stationarity by applying the ADF test. Secondly, we determine the order of the VAR process by using the SC, HQC information criteria and LR test. Thirdly, cointegration analysis, according to the Johansen (1988) procedure, is performed. Fourthly, and finally, tests for Granger causality are carried out on the selected VAR model. Note that this procedure has been applied once for the entire sample period, i.e. 1960:01 to 1998:9, and then separately for each of the two subperiods, 1960:01 to 1990:01 and 1990:02 to 1998:9, respectively.

When looking at the whole sample period of the monthly data, the ADF test results indicate that each variable is integrated of the same order one, i.e., I(1). For the time scales that are produced by the wavelet transformation, the test results indicate that all the series are to be considered as stationary variables. This is even the case for the separated sample periods. Using the above mentioned SC, HQC information criteria and LR test, we find the selected orders of the VAR process for each time scale. Repeating this procedure for each subsample, we find that the order of the VAR process remain the same. The same is right even when testing for the cointegration between the variables. The results from the Johansen test procedure have shown that the VAR systems for the integrated variables in the monthly data are cointegrated for all the sample periods.

When determining the manner of presentation, some account has to be taken to the results obtained. Our original intention was to present the results for the three time periods separately, but since the stationarity and cointegration results have shown to be the same for all the periods, and also, to make it easier to follow all the results from the study, we present our results in one overall table, Table I. The results from the different causality tests have shown to be fairly similar, and are also presented in this table. In what follows, we analyse the results starting by the monthly data and then the different wavelet's time scales.

When looking at the entire sample period and using the monthly data, the evidence indicates strongly feedback effect between the variables, i.e., that *LnS* and *LnR* are shown to strongly

TABLE I. Overall Estimated Results, P-values.

Time scale	LnS	LnR	VAR order	Nature of VAR	Test methods for causality	1960-1998			1960-1990			1990-1998		
						Results	Null Hypotheses		Results	Null Hypotheses		Results	Null Hypotheses	
							LnS LnR	LnR LnS		LnS LnR	LnR LnS		LnS LnR	LnR LnS
Monthly data	I(1)	I(1)	(5)	Cointegrated	Single-LR:	$LnS \Leftrightarrow LnR$	0.0000	0.0010	$LnS \Leftrightarrow LnR$	0.0000	0.0063	$LnS \Leftrightarrow LnR$	0.0004	0.0173
					Rao F-test:	$LnS \Leftrightarrow LnR$	0.0000	0.0004	$LnS \Leftrightarrow LnR$	0.0000	0.0025	$LnS \Leftrightarrow LnR$	0.0020	0.0009
					Bootstrap:	$LnS \Leftrightarrow LnR$	0.0000	0.0006	$LnS \Leftrightarrow LnR$	0.0000	0.0004	$LnS \Leftrightarrow LnR$	0.0023	0.0016
D1	I(0)	I(0)	(6)	Stationary	Single-LR:	$LnS \Rightarrow LnR$	0.0012	0.0687	inconclusive	0.1565	0.0753	$LnS \Rightarrow LnR$	0.0000	0.0981
					Rao F-test:	$LnS \Rightarrow LnR$	0.0018	0.0653	inconclusive	0.1565	0.0653	$LnS \Rightarrow LnR$	0.0001	0.0671
					Bootstrap:	$LnS \Rightarrow LnR$	0.0020	0.0630	inconclusive	0.1490	0.0680	$LnS \Rightarrow LnR$	0.0000	0.0820
D2	I(0)	I(0)	(4)	Stationary	Single-LR:	inconclusive	0.0700	0.9529	inconclusive	0.2980	0.9333	$LnS \Rightarrow LnR$	0.0166	0.8855
					Rao F-test:	inconclusive	0.0719	.9500	inconclusive	0.2988	0.9334	$LnS \Rightarrow LnR$	0.0155	0.7579
					Bootstrap:	inconclusive	0.0710	0.8900	inconclusive	0.2960	0.9380	$LnS \Rightarrow LnR$	0.0160	0.7930
D3	I(0)	I(0)	(2)	Stationary	Single-LR:	inconclusive	0.7520	0.5292	inconclusive	0.7705	0.6953	inconclusive	0.3298	0.6021
					Rao F-test:	inconclusive	0.8672	0.4599	inconclusive	0.7710	0.6960	inconclusive	0.6954	0.5153
					Bootstrap:	inconclusive	0.7080	0.5290	inconclusive	0.7910	0.7230	inconclusive	0.7140	0.5910
D4	I(0)	I(0)	(4)	Stationary	Single-LR:	inconclusive	0.1556	0.3736	inconclusive	0.3631	0.5758	$LnS \Rightarrow LnR$	0.0141	0.3869
					Rao F-test:	inconclusive	0.1300	0.3491	inconclusive	0.3638	0.5765	$LnS \Rightarrow LnR$	0.0149	0.5153
					Bootstrap:	inconclusive	0.1360	0.3340	inconclusive	0.3630	0.5660	$LnS \Rightarrow LnR$	0.0110	0.4750
D5	I(0)	I(0)	(5)	Stationary	Single-LR:	inconclusive	0.5296	0.3714	inconclusive	0.1311	0.5279	inconclusive	0.9733	0.4320
					Rao F-test:	inconclusive	0.5299	0.3717	inconclusive	0.1314	0.5283	inconclusive	0.3463	0.8320
					Bootstrap:	inconclusive	0.5900	0.5090	inconclusive	0.1660	0.5080	inconclusive	0.8720	0.7110

Granger cause each other. We obtained similar indications even for the separate subperiods, 1960:01 to 1990:01 and 1990:02 to 1998:9. This may mean that the decisions regarding the amount of spending and the decisions regarding the amount of taxes are taken simultaneously.

Looking at the very finest time scale, D1, the evidence from the whole sample period, indicates that *LnS* strongly Granger cause *LnR*. When analysing the first subperiod, however, the results indicate inconclusive causality nexus between the variables. When analysing the second subperiod, however, the results have shown that causality nexus may exist from *LnS* to *LnR*, which agrees better with Barro's tax hypothesis and those results found by Shukur and Hatemi-J (1998), using the quarterly data.

At the next finest time scale, D2, when looking at the entire period and the first subperiod, the results indicate inconclusive causality relation, while we find that *LnS* strongly Granger cause *LnR* during the second subperiod. The results from the second subperiod support those of D1 that are obtained from the same subperiod.

At the first intermediate time scales, the second intermediate time scales and the highest level of time scale, i.e., D3, D4, and D5, respectively, the results indicate inconclusive causality relation between the two variables in almost all cases. The only exception is for D4 during the second subperiod, the results in this case have shown that only *LnS* Granger causes *LnR*.

Note that when considering Figure 1 and 2, we can see how the wavelet transformations can successfully and clearly zoom out the high frequency variations in the data, which is not so clear when considering the original monthly data. A clear consideration of, for example, the second subperiod of the time scales D1 and D2 can show that the variations in the *LnS* series are much higher than the variations in the *LnR* series. This may give an alternative indication of the influence of the *LnS* on the *LnR*.

The results are fairly plausible. When analysing the monthly data, the relation between these variables indicate a feedback mechanism in all cases, which can imply that the decisions regarding the amount of spending and the decisions regarding the amount of taxes are taken simultaneously. On the other hand, when looking at the finest, and second intermediate scales, D1, D2 and D4, respectively, we generally find that strong causality nexus may exist from *LnS* to *LnR* during the second time period. This can imply that the decisions regarding the amount of spending precede the decisions regarding the amount of taxes, which agrees with Barro's tax hypothesis. In other words, at those time scales or at low frequencies, the political system in Finland, during the second subperiod, first decides how much to spend and then decides how much to bring in as revenue by taxes. This can be a result of the future plans of creating EMU.

7. CONCLUSIONS

In this paper, we empirically investigate the relation between government spending and revenue by using wavelet analysis that enables to separate out different time scales of variation in the data, i.e. we investigate the role of time scale in economic relations in terms of government spending and revenue.

When investigating the causal nexus of government spending and revenue in Finland, using monthly data and quarterly data, different results have been obtained. This may be due to the fact that there are several time scales involved in the relationship, and that the conventional analysis may be inadequate to separate out the time scale structured relationships between the variables.

Hence, we used the wavelet analysis in investigating the causal nexus of government spending and revenue in Finland during the period 1960:01 through 1998:09. Different test methods have been used to investigate the causality nexus between these variables. The most interesting result is that when looking at the finest, and second intermediate scales, D1, D2 and D4, respectively, tests results indicate that strong causality nexus may exist from *LnS* to *LnR* during the second time period, i.e., 1990 to 1998. This can imply that the decisions regarding the amount of spending precede the decisions regarding the amount of taxes, i.e. the political system in Finland, first decides how much to spend and then decides how much to bring in as revenue by taxes. The above result agrees with Barro's tax hypothesis and can be a result of the future plans of creating EMU.

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