

# A STABILIZED BANDWIDTH SELECTOR IN MULTIVARIATE KERNEL DENSITY ESTIMATION

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## Abstract

In recent years, most work in data-driven bandwidth selection methods in kernel density estimation has focused on the univariate case, while the multivariate case has been largely neglected. This may be due to the fact that it is technically more difficult to calibrate multivariate densities. Therefore, such multivariate methods represent a major growth area of research. In this paper, based on a random sample of size  $n$  from an unknown  $d$ -dimensional density  $f$ , the problem of selecting the bandwidth in kernel estimation of  $f$  is investigated. The main goal is, for all  $d$ , to propose a STS (stabilized bandwidth selector), which is a modified extension to general  $d$  of the selector of Chiu (1992) (who only dealt with the case  $d=1$ ). It is known that for all  $d$  the bandwidth selected by the LSCV (least squares cross-validation) has large sample variation. The proposed STS, as an improvement of LSCV, will reduce the variation of LSCV without significantly inflating its bias. The key idea of the STS is to modify the  $d$ -dimensional sample characteristic function beyond some cut-off frequency in estimating the integrated squared bias. It is shown that for every  $d$  and sufficiently smooth  $f$  and kernel, the STS is asymptotically normal with the optimal root  $n$  relative convergence rate and achieve the (conjectured) information bound. For  $d=2$  and  $3$ , a simulation study has been done, and the superior performance of the STS at practical sample sizes are clearly demonstrated.

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