A Measure to Judge the Plausible Change in Gini Index

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1. Introduction

It sometimes provokes a controversy in a society when the economists make a remark on the change in income distribution towards equality or inequality, no matter which direction it is. The judgement of this change is usually done through the change of the indicators like Gini Index. But what extent of margin or difference is required to say, for example, that the economy is heading for an equally distributed society?

The problem is that we don't have a definite procedure to judge the hypothesis. For example, Gini index is treated as a descriptive statistic in almost all the cases that it is difficult to say something definitive using just this index.

In this paper, we define a measure of plausibility to judge the change in Gini index, (1) when the form of income distribution is known, (2) when the form of income distribution is not known, using the likelihood method and the bootstrap method respectively. By using this measure, it will be easy to judge the plausible change in income distribution.

2. Use of Likelihood

Without loss of generality, we assume the Pareto distribution for income variable X_1 and X_2 for two periods respectively.

$$F(x_i) = 1 - \left(\frac{k}{x}\right)^{a_i}, \quad k > 0, a_i > 0, x \geq k, \quad i = 1,2$$

Then the ML estimator of Gini index is as follows.

$$\hat{G}_i = \frac{1}{2\hat{a}_i - 1}, \qquad \hat{a}_i = n\left[\sum_{j=1}^{n} \log\left(\frac{x_j}{\hat{k}}\right)\right]^{-1}, \quad \hat{k} = \min_j x_j, \quad i = 1,2$$

So, given data, we can calculate the likelihood of the true Gini indices G_1 and G_2 to lie in the region $G_1 < G_2$. If we standardize the likelihood of the full-region to unity, the measure of plausibility for the true Gini indices to lie in the region $G_1 < G_2$ is,

$$Pl(G_1 < G_2 \mid \hat{G}_1 < \hat{G}_2) = 1 - \int_0^{\frac{1+\frac{1}{\hat{G}_2}}{1+\frac{1}{\hat{G}_1}}} \frac{(2n_1)^{n_1}(2n_2)^{n_2}}{B(n_1,n_2)} \frac{x^{n_1-1}}{(2n_2+2n_1x)^{n_1+n_2}} dx$$

$$B(u,v) = \int_0^1 x^{u-1}(1-x)^{v-1} dx$$

This is easily calculated for every pair $\hat{G}_1$ and $\hat{G}_2$, and applicable to any other income distribution..

3. Use of Bootstrap Method

In most cases, we can't specify the form of the income distribution. Even in this case, we can construct a measure of plausibility similar to the one defined above.

After we calculate the Gini index $\hat{G}_1$ and $\hat{G}_2$ from the raw data, we recalculate $\hat{G}_1'$ and $\hat{G}_2'$ from bootstrap sampling. The inequality relation of $\hat{G}_1'$ and $\hat{G}_2'$ is not necessarily the same as that of $\hat{G}_1$ and $\hat{G}_2$, if the values $\hat{G}_1$ and $\hat{G}_2$ is very close or structural change is weak. So we define a measure of plausibility, by collecting the bootstrap Gini indices, as the proportion of the number of bootstrap Gini indices to hold the same inequality relation as the original $\hat{G}_1$ and $\hat{G}_2$, to the total number of bootstrap sampling. We call it the 'Plausibility Measure'.

4. Some Properties of the Plausibility Measure

We find that the Plausibility Measure has some good properties with respect to the stability and the sensitivity, as a result of executing a set of simulations

5. Conclusions

We defined the Plausibility Measure to express a kind of degree of belief about the change of Gini index. But this method can be applied to any other statistical indicators. We propose to use this kind of measure to make the reliable statement based on statistical indicators.

Résumé

Il arrive très souvent que l'on ait une difficulté à tirer une conclusion sur le changement de la distribution des revenus entre deux périodes sur la base des indices économiques. Nous proposons une mesure de plausibilité du changement de l'indice Gini à l'aide de la méthode bootstrap afin d'émettre une véritable conclusion.