On Some Association Measures in Bivariate Distributions and Their Relationships

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ABSTRACT: In this paper, we study three time dependent measures of association, viz. the odds ratio, conditional mean residual life and the conditional probability. We examine conditions under which these measures exceed unity. We also obtain a class of distributions for which the components of the hazard gradient (mean residual lives) are proportional. Finally, some examples are provided to illustrate the results.

INTRODUCTION

The most common measures for determining the relationship between two variables are the correlation coefficient, Kendall's tau and Spearman's rho. Other measures of association have been defined by Clayton and Cuzick (1985) and Hougaard (1986). But all of these measures do not allow time dependence in the association structure. Oakes (1989) presented a time dependent association measure interpreted as an odds ratio and examined its properties for a larger class of bivariate survival functions. Anderson et al (1992) defined two other time dependent measures, one based on conditional life expectancy and the other is based on conditional probability and studied some of their properties. They based their investigation on a particular form of survival function given by equation (1) of their paper and applied their results to the Danish twin data. They concluded that in twin study or a study of lifetimes of a parent and a child, the conditional expected residual life measure provides an appropriate summary of association for demographic and acturial analyses designed to predict longevity for individuals and small groups. In a study of breast cancer in women, the conditional probability measure describes the dependence between age at breast cancer diagnosis and age at birth of first child. This dependency summary could assist in family planning decisions.

It is, therefore, important to study these measures in more detail and to investigate some some relationships between them. In this paper, we have obtained the class of distributions for which these measures indicate a positive association between the two variables and provided several

examples. More specifically, we study the odds ratio measure $q(x, y)$ and investigate conditions under which $q(x, y) > 1$ or $q(x, y) = 1.$ We also obtain a class of distributions for which the two failure rates involved in the definition of $q(x, y)$ or the two failure rates involved in the definition of hazard gradient are proportional. We also study the measure $f_1(x, y),$ which is based on conditional life expectancy and investigate its connection with $q(x, y).$ We examine as to when the two components of the mean residual lives are proportional. Finally, the measure $y(x, y)$ based on the conditional probability, is presented and its relationship with the other measures is investigated.

Because of the page limitations, the details will be given else where.

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