

Multivariate Log – Normal Distribution

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1-Introduction

Multivariate distributions have important role in economics and statistics. Since variables in economics, psychology, and reliability are positive; multivariate positive distributions should be considered. Johnson, and Kotz (1972) cited some multivariate positive distributions such as multivariate Gamma, Beta, Pareto, F distribution which have very complicated forms and are not easy to use in Economical and Reliability investigations.

Jeevavand (1997) drive the Bayesian estimate of $P(X_1 < X_2)$ in bivariate Pareto distribution. Tong (1980) developed the area of probability inequalities in multivariate distributions. Tarmast (1997) used multivariate normal distribution to obtain bounds for the multiple integral $P(B < X < C)$.

Here, multivariate log - normal distribution is defined and its mean and covariance matrix are obtained and their estimates are calculated. The application of the multivariate log - normal distribution in reliability is mentioned.

2-Multivariate Log-Normal Distribution

Let $X = [X_1, X_2, \dots, X_p]$ be a p-component random vector having multivariate Normal distribution with mean v and covariance matrix $D = (d_{ij})$. Now we use the transformation $Y_i = \exp(X_i)$ and define $Y = [Y_1, Y_2, \dots, Y_p]$. The density of Y is multivariate log - normal distribution and has the following form:

$$f_Y(y) = (2\pi)^{-p/2} |D|^{-1/2} [y_1, y_2, \dots, y_p]^{-1} \text{Exp}[-(Lny - v)'D^{-1}(Lny - v)/2] \quad 0 < y_i < \infty \quad (1).$$

Where $Lny = [Lny_1, Lny_2, \dots, Lny_p]$ is a p-component column vector and $y_i = \exp(x_i)$.

3-Mean and Covariance Matrix

Let Y be a p-component vector and have multivariate log - normal distribution. Mean of Y is :

$$\mu = E(Y) = [\mu_1, \mu_2, \dots, \mu_p] \quad \mu_i = \exp(v_i + 0.5d_{ii})$$

Where d_{ii} is ⁽²⁾ ith. Diagonal element of matrix D. Covariance matrix of Y is :

$$\Sigma = E[(Y - \mu)(Y - \mu)'] = (\sigma_{ij}) \quad \sigma_{ij} = [\exp[(v_i + v_j) + (d_{ii} + d_{jj})/2]] * [\exp(d_{ij}) - 1] \quad (3).$$

Where d_{ij} is the ijth. Element of D. It is clear that if X_1, X_2, \dots, X_p are independent, then Y_1, Y_2, \dots, Y_p are also independent and vice versa.

4-Estimation

Let $Y^{(1)}, Y^{(2)}, \dots, Y^{(n)}$ be a random sample of p-component vectors having multivariate log-normal distribution and $X^{(1)}, X^{(2)}, \dots, X^{(n)}$ be the corresponding random sample of vectors having multivariate normal distribution respectively. The maximum likelihood estimators of μ and Σ are considered and obtained. To find these estimators the maximum likelihood estimators of v_i, d_{ii}, d_{ij} should be found. Suppose $\underline{X}_i, S_{ii}, S_{ij}$ are the maximum likelihood estimators of v_i, d_{ii}, d_{ij} respectively.

$$\bar{X}_i = (\sum X_{ij})/n \quad S_{ii} = \sum (X_{ij} - \bar{X}_i)^2 \quad S_{ij} = \sum (X_{ik} - \bar{X}_i)(X_{jk} - \bar{X}_j) / n$$

(4).

Now, the maximum likelihood estimators of μ_i, σ_{ij} can be found.

$$\hat{m}_i = \exp[\bar{x}_i + S_i^2/2] \quad \hat{S}_{ij}^2 = [\exp(2\bar{x}_i + S_i^2/2) * [\exp(S_i^2) - 1]]$$

(5)

$$\hat{S}_{ij} = [\exp[(\bar{x}_i + \bar{x}_j) + (S_i^2 + S_j^2)/2]] * [\exp(S_{ij}) - 1]$$

Where \hat{m}_i is the i th. Element of the maximum likelihood estimator of m and \hat{S}_{ij} is the ij th. element of the maximum likelihood estimator of Σ .

5-Application

Let $Y = [Y_1, Y_2, \dots, Y_p]$ be a p -component random vector having multivariate log-normal distribution and $B = [B_1, B_2, \dots, B_p]$ and $C = [C_1, C_2, \dots, C_p]$ be two given finite positive constant vectors such that $B_i \leq C_i$, then the upper and lower bound for $p(B \leq Y \leq C)$ can be found:

$$p(B \leq Y \leq C) = p(\ln B \leq \ln Y \leq \ln C) = P(B_0 \leq X \leq C_0).$$

(6).

Where X has multivariate normal distribution and B_0 and C_0 are given constants. The upper and lower bounds for (6) can be found. See Tarmast(1977).

6-BIBLIOGRAPHY

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7-SUMMARY

Let X be a random vector have a multivariate normal distribution. Then, $Y = \exp(X)$ is a random vector having multivariate log-normal distribution. The density distribution of Y is obtained and mean and covariance matrix of Y are estimated and the application of the distribution is mentioned.