

# The Necessary and Sufficient Condition for 3-Way Balanced Multi-level Rotation Sampling Designs

YouSung Park

*Assistant Professor, Department of Statistics, Korea University*

*Anam-dong 5-1, Sungbuk-gu, Seoul 136-701, Korea. [yspark@mail.korea.ac.kr](mailto:yspark@mail.korea.ac.kr)*

JaiWon Choi

*CDC, National Center for Health Statistics, Hyattsville, MD 20782, U.S.A. [jwc7@cdc.gov](mailto:jwc7@cdc.gov)*

NaYoung Kim

*Department of Statistics, Korea University. [nora@mail.korea.ac.kr](mailto:nora@mail.korea.ac.kr)*

## 1. Introduction

Rotation sampling designs may be classified into two categories by the frequency that a selected sample unit appears in the sample. The first type uses the same sample unit for the entire life of the survey. The second type uses the sample unit only for a fixed number of times, and we call it semi-rotation sampling design. In the first type, we systematically select some of the rotation groups and then use all sample units in each selected group for the monthly sample. However, in the semi-rotation design all rotation groups are included in the sample, but the sample unit in a certain group is replaced by a new unit from the same group following a specific rotation plan.

Bailer (1975) discussed the bias arising from different interview times, and Cantwell and Caldwell (1998) demonstrated biases from unbalanced rotation groups and recall times and showed that those biases overwhelm the mean squares error. To overcome unbalancing problem, we consider a class of semi multi-level rotation designs which is a general version of the semi one-level rotation design (Park, Kim and Choi, 2000).

We illustrate the multi-level rotation design with the “3-level” 4-8-4 design (Figure 1). In the 3-level 4-8-4 rotation design, a sample unit is in the sample every third month for a total of 4 times, gives no information for the next 8 months and finally returns to the sample every third month for another 4 times. The notations  $(\alpha, g)$  in Figure 1 are the symbols for the unit  $\alpha$  in the group  $g$  and  $u_i$  is for the unit  $\alpha$  interviewed in the  $i$ -th time (or turn). The symbols “i” and “ii” above the sample unit  $u_i$  denote the same sample unit which provides the information of the previous 2 months. Figures 1-(i), (ii) and (iii) show the rotation patterns for the recall times 0, 1 and 2, respectively. These three pictures are obtained from Figure 1 when ignoring  $\alpha$ . We observe that the picture of each recall time is balanced in 2-ways, horizontally and perpendicularly, where the perpendicular balancing is done for any span of 8 months. Figure 1

is a typical example of a multi-level rotation design which is balanced in 3-ways by interview time (horizontally in each picture), by interview time (perpendicularly in each picture) and by recall time (the same pattern in all three picture (i), (ii) and (iii)).

Our main point in this paper is to show how to construct the 3-way balanced multi-level rotation design to reduce the design bias and to present the necessary and sufficient conditions for the 3-way balancing. We extend the previous 2-way balanced one-level rotation design (Park, Kim and Choi, 2000) to the 3-way balanced multi-level rotation design, and discuss the properties of the 3-way balanced design as well as its rotation pattern. We also provide the necessary and sufficient condition for a multi-level rotation design to be balanced in 3-ways and we present an algorithm to construct 3-way balanced multi-level rotation designs and illustrate how to use this algorithm.

Figure 1: The 3-way balanced multi-level 4-8-4 design.

$\alpha$	1								2								3								4								5								6									
$g$	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2
$t$	$u_8$																																																	
$t+1$																																																		
M $t+2$																																																		
O $t+3$																																																		
N $t+4$																																																		
T $t+5$																																																		
H $t+6$																																																		
$t+7$																																																		
$t+8$																																																		
$t+9$																																																		
$t+10$																																																		
$t+11$																																																		
$t+12$																																																		
$t+13$																																																		

(i) $recalltime = 0$									(ii) $recalltime = 1$									(iii) $recalltime = 2$								
Month	Rotation Groups								Month	Rotation Groups								Month	Rotation Groups							
	1	2	3	4	5	6	7	8		1	2	3	4	5	6	7	8		1	2	3	4	5	6	7	8
$t$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$t$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$t$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$
$t+1$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$t+1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$t+1$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$
$t+2$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$t+2$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$t+2$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$
$t+3$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$t+3$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$t+3$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$
$t+4$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$t+4$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$t+4$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$
$t+5$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$t+5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$t+5$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$
$t+6$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$t+6$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$t+6$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$
$t+7$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$t+7$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$t+7$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$
$t+8$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$t+8$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$t+8$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$
$t+9$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$t+9$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$t+9$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$
$t+10$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$t+10$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$t+10$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$
$t+11$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$u_4$	$t+11$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$t+11$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$
$t+12$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$u_7$	$t+12$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$t+12$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$
$t+13$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$u_2$	$t+13$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$	$u_5$	$t+13$	$u_5$	$u_2$	$u_7$	$u_4$	$u_1$	$u_6$	$u_3$	$u_8$

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