The Minor Works of Thomas Bayes

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1. Introduction

In his *Arithmetical Books* of 1847 de Morgan stresses the importance of the study of 'the minor and secondary phenomena of the progress of mind', and notes that such a study cannot take place 'until the historian has at his command a readier access to second and third rate works in large numbers'. It is in acceptance of these views that attention is paid here to some minor works of Thomas Bayes. For these works, though clearly not of the gravity and interest of the 1763 *Essay towards solving a Problem in the Doctrine of Chances*, nevertheless shed light on more important 18th century works.

2. Divine Benevolence

Bayes's *Divine Benevolence, or, an attempt to prove that the principal end of the divine providence and government is the happiness of his creatures* was printed in 1731 as a rebuttal to John Balguy's *Divine Rectitude* of 1730, and it was in turn followed in 1734 by Henry Grove's *Wisdom, the first Spring of Action in the Deity.* Like Bayes, Balguy and Grove each sought a single principle to which God's moral principles could be ascribed; and although their conclusions were different (Bayes's fundamental attribute of *benevolence* being *rectitude* in Balguy's tract and *wisdom* in Grove's), Balguy and Grove both discussed the place of benevolence in their schemes.

At the start of his second section Bayes lists two points (these points being essentially his main thesis) that he proposes to prove, viz.

> 1. That God in his acts of creation and providence had a regard to the happiness of his creatures, and that he is really benevolent towards them. And, 2. That we have no reason to suppose that he

> is in his actions towards them influenced by any other principles; at least, by any other, that are not entirely coincident with, or perfectly subordinate to this. [p. 20]

These, taken together, seem to provide what one might call an 'existence and uniqueness' proof for the principle of benevolence.

Balguy gave several reasons, distinct from a regard for His creatures' happiness, for God's conduct. The first of these, discussed in the fourth section of Bayes's tract, is concerned with the regard God has for *beauty* and *order* in his works. Bayes does not deny that God indeed has such regard: rather, 'I only deny that this was an end of his acting distinct from the happiness of his creatures, and on the contrary affirm, that it was a regard to their felicity which was the reason why he has made and disposes of all things in so orderly and beautiful a manner' [p. 35].

It has been suggested before (see Stigler (1983)) that Bayes and David Hartley were acquainted. Such a suggestion perhaps gains additional credence on our noting that in the latter's *Observations on Man* of 1749, considerable attention is given to *benevolence*. Indeed, one finds here the repetition of several of the opinions displayed in Bayes's treatise, *inter alia*: the emphasis on God's benevolence at the expense of detailed consideration of His rectitude or wisdom; the arguments for such benevolence are found by Hartley to be derivable from the happiness observable in sentient beings; the presence of misery is not to be seen as destructive of the evidences for the divine benevolence; God is infinitely benevolent; and malevolence is excluded.

3. Introduction to the Doctrine of Fluxions

Controversy over the concepts and implementation of the fluxionary calculus abounded in the early 1700s. Among these controversialists was George Berkeley, who in 1734 published *The Analyst; or, a Discourse Addressed to an Infidel Mathematician*, in which he criticized the methods of the fluxionary and the differential calculi and the ontological status of the objects therein considered. Many were the replies that this diatribe provoked, including Bayes's *An Introduction to the Doctrine of Fluxions* of 1736.

Bayes's response to Berkeley was concerned more with the logical analysis of the theory of prime and ultimate ratios than with the methods of the fluxionary calculus or moments (or with matters theological), and it is with this in mind that the tract should be read.

After stating some fundamental postulates, definitions and axioms, Bayes essentially proves the uniqueness of the prime and ultimate ratios. It is perhaps worth comparing his proofs with those used today to show the uniqueness of the left- and right-hand limits: the proofs are seen to be remarkably similar, provided that one does not interpret Bayes's 'vanishing' of flowing quantities too literally.

Bayes now sets down nine propositions in which various properties of fluxions are derived. For example, in Proposition V he shows that the fluxion of a sum of two fluents is the sum of the fluxions, provided that the fluents both increase or decrease together; in Proposition VI this last proviso is removed; and in Proposition VII the fluxion of Az is shown to be $A\dot{z}$ (with A being determinate). The eighth proposition states that the fluxion of x^2 is $2x\dot{x}$, two proofs being given (the first seems correct, but the second needs some attention). The ninth proposition is concerned with the fluxion of the product of two flowing quantities. As a corollary Bayes shows that $\dot{z} = \dot{x}y + \dot{y}x$ if fluxions of decreasing quantities are taken to be negative.

In the third section of the essay Bayes replies to some of Berkeley's criticisms of Newton's arguments. For instance, he proposes to show, following Newton, that the fluxions of x and x^n are in the ratio $1 : nx^{n-1}$. First he expands the binomial $(x + o)^n$, and then he concludes that the synchronal augments o and $n\,o\,x^{n-1} + ((n^2 - n)/2)\,o^2\,x^{n-2} +$ *&c.* are to one another as $1 : nx^{n-1} + ((n^2 - n)/2)\,o\,x^{n-2} +$ *&c.* So far there can be no disagreement. The difficulty arises in his now saying 'Let now these augments vanish', an assumption that is needed to conclude that the fluxions of x and x^n are in the ratio $1 : nx^{n-1}$. The problem is of course avoided in modern practice by the use of limits, an approach that was not open to Bayes.

4. On a semi-convergent series

In the half-century preceding the publication of *A Letter from the late Reverend Mr. Thomas Bayes, F.R.S. to John Canton, M.A. and F.R.S.* in 1764, infinite series had been examined in a number of papers in the *Philosophical Transactions*. Yet the *Letter* was in a sense unique, addressing itself to the divergence of $\sum_1^z \log k = (1/2)\log 2\pi + (z + (1/2))\log z - \mathfrak{S}$, where

$$\mathfrak{S} \;=\; z - \frac{1}{12z} + \frac{1}{360z^3} - \frac{1}{1260z^5} + \frac{1}{1680z^7} - \frac{1}{1188z^9} + \&c.$$

By examining the rule for the formation of the coefficients and noting essentially that the coefficients of the nth and $(n+1)$th terms on the right-hand side in $\mathfrak{S}$ (y_n and y_{n+1} say) satisfy $|y_{n+1}/y_n| > (n-1)(2n-3)/(6n+3)$, $n \geq 3$, Bayes shows that 'at length the subsequent terms of this series are greater than the preceding ones, and increase in infinitum, and therefore the whole series can have no ultimate value whatsoever' [p. 270].

5. Conclusion

I find it difficult, as one who is not theologically trained, to pass an authoritative judgement on the relative merits of the tracts by Balguy, Bayes and Grove: it is unclear, at least to me, who, if any, emerged the victor in the 'celebrated controversy'. Perhaps the most we can say is that we are, by the time we have finished reading the tracts, if not wiser, at least better informed on God's attributes and the motives guiding Him.

Whether Bayes's reply to Berkeley can be regarded as a success is arguable. Published opinions vary, the most cautious perhaps being that of Cajori, who writes 'The pamphlet [by Bayes] of 1736 represents a careful effort to present an unobjectionable foundation of fluxions' (1919, p. 157). Some of Bayes's arguments seem to foreshadow aspects of limit theory later to be used in the rigorization of the calculus; but they fail to convince today, adequate though they might have been at the time.

The *Letter* on the Stirling-de Moivre series is a serious bit of mathematical work, containing an argument that would, at some time or other, have to have been made.

REFERENCES

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RESUME

Nous discutons dans cet article trois oeuvres mineures de Thomas Bayes en les comparant avec d'autres travaux du XVIII[e] *siecle.*