Log-Periodogram Regression in the Presence of Trends

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1. Introduction

Long-memory time series have represented a popular area of research in econometrics as well as in statistics in recent years because of their applicability in many sciences. Besides its applicability for hydrologic time series long-range dependence is also found in the second moments of many stock returns. Long-range dependence or long-memory means that the correlation of a time series decays hyperbolically and thus the correlations of a long-memory time series are not summable.

In general long–range dependence can be defined by the shape of the spectral density of the process. Let $f(\lambda)$ be the spectral density of the stationary process X_t. Then X_t is said to exhibit long-memory, if

$$f(\lambda) \sim_{|\lambda|\to 0} L_2(\lambda)|\lambda|^{-2d}, \quad d \in (0, 1/2),$$

where $L_2(\lambda)$ is slowly varying for $|\lambda| \longrightarrow 0$.

In the recent years it is a much debated problem, if the long-memory structure observed in economic time series is spurious long-memory and thus the result of any other phenomenon as structural breaks or more general trends. Beginning with Bhattacharya et al. (1983) it is shown that deterministic trends added to a short-memory process can produce spurious long-memory. Giraitis et al. (2000) proved that rescaled-range based test on long-range dependence cannot distinguish small trends and long-range dependence. Krämer/Sibbertsen (2000) showed that tests on structural breaks reject the null hypothesis of no structural break with probability tending to one if there is long-memory in the data. Thus it is still an unsolved problem to distinguish arbitrary trends and long-range dependence. For an overview see Sibbertsen (2001). Künsch (1986) proved that the periodogram of a time series behaves different for monotonic trends and long-range dependence. Motivated from this result we consider in this paper the

behaviour of a periodogram based estimator of the memory parameter in the case of trending data.

2. Log-periodogram regression in the presence of deterministic trends

Using the special shape of the spectral density of a long-memory process an estimator for the memory parameter d is defined by the least-squares estimation of d in the following linear regression model:

$$\ln I_X(\lambda_i) = c - d \ln \left\{ 4 \sin^2 \left(\frac{\lambda_i}{2} \right) \right\} + \epsilon_i, \quad i = 1, \ldots, m.$$

Here $I_X(\lambda)$ denotes the periodogram of the process X and λ_i are the harmonic ordinates, ϵ_i is an error term of iid exponentially distributed zero mean random variables and c is the intercept.

We consider the model $X_t = f(t) + Y_t$, where $f(t)$ is a deterministic trend and Y_t is a short-memory process. It turns out that under this model the log-periodogram estimator is positively biased depending on the shape of the trend. For small trends the bias is negligible. Asymptotic normality of the estimator can no longer be proved under this model.

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RESUME

Le phénomène constaté que des tendances dans les données peuvent produire une mémoire longue apparente a fait l'objet de nombreuses discussions au cours des dernières années. Nous étudions donc dans ce travail les conséquences de tendances sur des estimateurs fondés sur le périodogramme logarithmique pour le paramètre de mémoire. Il s'avère qu'une déformation de l'estimateur dépendant de la forme de la tendance apparaît. Dans ce modèle, il n'est plus possible de démontrer la normalité asymptotique de l'estimateur.