

Conditional Analysis of Three Way Contingency Tables by Metropolis Walks

Ryuichi Sawae

Okayama University of Science, Applied Mathematics

1-1 Ridai-cho

Okayama, Japan

sawae@xmath.ous.ac.jp

Toshio Sakata

Kyushu Institute of Design, Industrial Design

4-9-1 Shiobaru Minami-ku

Fukuoka, Japan

sakata@kyushu-id.ac.jp

1. Three Way Contingency Tables

Recently we have calculated the Groebner basis of three way contingency table $4 \times 4 \times 4$. The number of its reduced Groebner basis turns out to be 329543, and the unique base, which has highest degree, is as follows.

$$x_{111}x_{114}x_{133}^2x_{144}x_{213}^2x_{224}^3x_{242}x_{323}^2x_{332}x_{334}x_{341}^2x_{414}x_{421}^2x_{422}x_{434}x_{444} - \\ x_{113}^2x_{134}^2x_{141}x_{214}^2x_{222}x_{223}^2x_{244}x_{321}^2x_{333}x_{342}x_{344}x_{411}x_{424}^3x_{432}x_{441}$$

For real data we apply the method (see and consider testing the null hypothesis of no three factor interaction for $4 \times 4 \times 4$).

2. Metropolis Walks on the $4 \times 4 \times 4$ table

In JKCS-2000 we reported the following simulation results for the data of the attitude for abortion in England on the $4 \times 4 \times 4$ table, (see Diaconis et al., the $3 \times 3 \times 3$ table data of the attitude for abortion in USA). But, at that time we could not obtain the whole set of Groebner basis for this $4 \times 4 \times 4$ table. Because it was under calculation. So, using about 20% of them we performed the simulation of 20,000,000 repeatetion. As for the simulation we obtained the p-value 0.068 and the value was stable under the simulation. From this we concluded that the three factor interaction might exist in this data.

As real data we used the data below from British Social Attitude Survey. They measured 9 variables, and the variables used here are three, that is, age, religion and attitude. The each category of age stand for under 25, 26 – 40, 40 – 55, and over 56 respectively. The categories A, B, C and D of Religion stand for Roman Catholic, Church of England, Others, and No religion

respectively. The attitude "++" means "strong support" and "--" means "strong objection" and others correspond to the modest one.

Age 1

	Attitude for abortion			
Religion	--	-	+	++
A	3	10	2	1
B	1	3	0	0
C	0	1	2	1
D	2	13	7	6

Age 2

	Attitude for abortion			
Religion	--	-	+	++
A	0	1	4	7
B	0	5	4	3
C	4	5	9	6
D	0	6	11	9

Age 3

	Attitude for abortion			
Religion	--	-	+	++
A	0	0	0	0
B	1	13	4	6
C	4	19	2	3
D	0	4	5	7

Age 4

	Attitude for abortion			
Religion	--	-	+	++
A	2	2	0	0
B	0	8	5	7
C	8	16	7	1
D	0	0	3	1

In this time, we will perform the Metropolis walk simulation with all the reduced Groebner basis for the above data($4 \times 4 \times 4$ table).

REFERENCES

Sawae, R. and Sakata, T. (2000). Echelon form and conditional test for three way contingency tables. *JKCS-2000*, 233-238.

Sawae, R., Sakata, T. and Teppei A. (1999). Algebraic algorithms for finding a Groebner basis of the ideal generated by contingency tables, *ORDAL'99 Montpellier 3rd International Conference on Orders, Algorithms and Applications, PreProceeding*, 230-231.

Diaconis, P. and Strumfels, B. (1998). Algebraic algorithms for sampling from conditional distributions. *A.S.*, 363-397.

Sakata, T., Nomakuchi, K. and Hayashi, T. (1997). Generalized shift test method and Metropolis walk *Bulletin of Information and Cybernetics*, 1-13.

RESUME

For $4 \times 4 \times 4$ three-way contingency tables we construct the conditional test of no three factor interactions by Metropolis walks. Simulation results for some real data is reported. The Metropolis walk was generated by the Markov basis derived from the Groebner basis which was calculated by Lattice basis method and its basis was calculated by the echelon form of the integer matrix related to the contingency tables.