Test for Equality of Coefficient of Variation of *K* Populations

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1. Subtitle

Test for coefficient of variation

Abstract: The coefficient of variation (C.V.) is widely used in agriculture, biology and allied diciplines. In this paper, a test is proposed for equality of C.V.s of k populations assuming normality. Type I robustness of the proposed test is studied through simulations. The test is fairly robust when the sample sizes are equal.

1. Test statistic: Consider independent samples of sizes $n_1, \ldots, n_k$ from k normal populations with mean $\mu_1, \mu_2, \ldots, \mu_k$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2$

Let $\theta_i = \dfrac{\sigma_i}{\mu_i}, \quad i = 1,2,\ldots,k.$

The hypothesis of interest is H: $\theta_1 = \theta_2 = \ldots = \theta_k$

Following the principle of Wald test the test statistic for testing H is

$$W = h'(\hat{\theta})[H(\hat{\theta})V(\hat{\theta})H'(\hat{\theta})]^{-1}h(\hat{\theta})$$

where,

$$\underset{(k-1)\,x\,k}{h(\theta)} = [\theta_1 - \theta_2, \theta_1 - \theta_3, \ldots, \theta_1 - \theta_k]'$$

$$\underset{(k-1)\,xk}{H(\theta)} = \left(\frac{\partial}{\partial \theta_j} \left(\theta_1 - \theta_j \right) \right)$$

$$\underset{kxk}{V(\theta)} = \text{diag}\left[V(\hat{\theta}_1), V(\hat{\theta}_2), \ldots, V(\hat{\theta}_k) \right]$$

$\hat{\theta}_1$ = m.l. estimator of θ

$$V(\hat{\grave{e}}_1)=\grave{e}_i^4+\frac{1}{2}\grave{e}_i^2,\ i=1,2,\ldots,k$$

The asymptotic distribution of the test statistic under H is that of a central chi-square random variable on (k-1) degrees of freedom.

2. Results and Discussion

The adequacy of the chi-square approximation and the type I robust ness of the test is studied through simulation. Balanced and imbalanced group sizes are considered when the number of groups are 2, 3, 5, 10 and the average sample sizes per group are 5, 10, 15 and 20. For robustness, the distributions considered are log normal and gamma and in the balanced set up correlated observations are also included from equi-correlated multi variated normal distributions. A sample results is given below for balanced set up.

Type I error rates $\alpha = 0.05$

Independent samples					Correlated observations k=5		
k	N	normal	Log-normal	gamma	ρ	N=5	N=20
3	5	0.155	0.171	0.159	-0.1	0.151	0.072
3	20	0.086	0.079	0.068	0.1	0.172	0.079
10	5	0.296	0.292	0.247	0.5	0.114	0.022
10	20	0.106	0.102	0.114	0.7	0.023	0.012

3. Conclusions:

1. Chi-square approximations for the test statistic is fairly accurate when the sample sizes are equal and $k \leq 5$. The type I error rates exceed nominal level α for the in balance cases considered.
2. When $k \leq 5$, for balance case, the test is robust for log normal and gamma populations.
3. In the in balance setup the test maintains type I error rates for log normal distribution.
4. The test is type I robust, when the observations are positively correlated. Because of the problem of singularity of the multivariate normal distribution, all combination of negative correlation could not be worked out and the results are inconclusive.

REFERENCE

Cox D.R. and Hinkley D.V. (1974) *Theoretical statistics*. London, Chapman and Hall

RESUME

Dr. K. Arun Rao is currently working as Reader in Statistics, Mangalore University, Mangalagangothri, India. He obtained his Ph.D. in statistics from Karnatak University, Dharwad, India. He was visiting professor in the University of Botswana during 1998-99. He has got 22 publications in refereed journals. His field of interest are, statistical inference, multi-variate analysis, linear and non- linear regression models, design of experiments and survival analysis.