AOQL: The Exact Version

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1. Introduction

The Average Outgoing Quality Limit (AOQL) sampling system was originally developed by Dodge & Romig (1959) for Bell Telephone System in the thirties. Van Batenburg & Kriens (1988) e.g. observed that the statistical derivation by Dodge and Romig was not completely convincing and they presented a modification. This modified version of the AOQL-method was called the EOQL-method (Expected Outgoing Quality Limit). Simons et al. (1989) developed an algorithm for finding the sample size and the critical level of items in error in the sample for the EOQL-method. However, this algorithm uses Poisson approximation for the hypergeometric distribution involved.

2. The AOQL-sampling procedure

Consider a population of size N consisting of M bad items and $N - M$ good items. A sample of size n is taken from the population and the number of bad items K is determined in the chosen sample. If K exceeds k_0 selected on beforehand, then the population is rejected and all items in the population will be inspected. Bad items are always corrected. If k does not exceed k_0, then only the bad items in the sample are corrected. So, after inspection the quality of the population has improved unless no bad items are found in the sample. The fraction of bad items after inspection equals p_A. The sample size n is determined in such a way that the expected fraction of bad items after inspection in the population does not exceed a predefined level P_l. Obviously, K follows a hypergeometric distribution with parameters n, M and $N - M$.

The expected fraction of bad items after inspection is given by

$$\pi_A = \mathrm{E}(p_A) = \sum_{k=0}^{k_0} \frac{M-k}{N} \mathrm{P}(K=k),$$

which differs from the expression given by Dodge and Romig. The function π_A depends on four parameters, namely k_0, n, M and N. We consider k_0 to be fixed. We look at π_A as a function of n, M and N and write $\pi_A(n, M, N)$.

3. Some results

This paper presents a very efficient and appealing algorithm for finding the sample size n for given k_0 that fully exploits the underlying hypergeometric distribution. This algorithm uses the facts that $\max_M \pi_A(n, M, N)$ is a strictly monotonic decreasing function of n and that $\pi_A(n, M, N)$ is a unimodal function of M. The paper proves these results. Some very nice characteristics of the hypergeometric distribution also appear. One of these characteristics is that $\mathrm{P}(K \leq k_0)$ is a logconcave function of M and strictly logconcave on $R_M = \{M | k_0 - 1 \leq M \leq N - n + k_0 + 1\}$. For generating tables for the AOQL-procedure the following result is very useful. Let n and k_0 be given and let $M^*(N)$ be the smallest solution such that $\pi_A(n, M^*, N) = \max_M \pi_A(n, M, N)$, then

$$M^*(N) \leq M^*(N+1) \leq M^*(N) + 1.$$

If we compare the sample sizes of the original AOQL-procedure using the Poisson-approximation with the method presented in this paper, then we see that the original method underestimates the optimal sample size. This results in expected fractions of errors after inspection which are too high. The original method of Dodge and Romig does not reject often enough. Of course the differences are most eminent for small sample sizes ($\pm < 1000$).

REFERENCES

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RESUME

Cet article donne une version révisée et améliorée de la méthode AOQL, développée par Dodge et Romig. Cette version est basée sur la distribution sous-jacente vraie, c.a.d. la distribution hypergéométrique.