Comparison of Trapezoidal and Triangular Membership Functions of Two Servers Queuing Models with Fuzzy Service Times

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1. Introduction

Traditional queuing models need to be defined by exactly defined distributions. However, for the queuing applications it is difficult define an exact distribution for the service rate. To express the service rate, mostly the linguistic expressions are used, such as "service is fast", "service is slow" or "service is not fast enough". Due to the fact that, it is more realistic to say that the service rate is more possibilistic than probabilistic. This structure of queuing system can be solved by fuzzy queuing models.

In this study, two servers queuing models with fuzzy service rate are investigated. Two types of membership functions; trapezoidal and triangular membership functions are defined.

2. M/F/2 Queues

One server fuzzy queuing systems had been studied by Li and Lee, Negi and Lee. In such systems interarrival times, service times or both are defined by fuzzy numbers. In this study, M/F/2 queuing systems are defined. In M/F/2 systems arrivals are Poisson and service times are fuzzy numbers. By using α-level sets this system is reduced to a crisp queuing system. To derive the queue characteristics, the mean and the standard deviation of each α-level sets is calculated. At each α-level, service times are presented by uniform distribution and they have the interval of confidence as $[x_{\alpha 1}, x_{\alpha 2}]$. The membership function is firstly defined as trapezoidal, secondly as triangular. In the model, channels are assumed as independent and queue discipline is first come first served.

3. An Example

Simulation is used for an exampling for a M/F/2 system. Arrivals are assumed to be Poisson with 0.02 arrival rate. α-levels are determined as 0, 0.05, 0.10, ..., 1. At each level, random trapezoidal and triangular fuzzy service time is generated by simulation. For both fuzzy numbers, trapezoidal and triangular membership functions are formulated. Queuing characters are calculated for both models. Results are shown at Table 1.

4. Conclusion

At Table 1, L, Lq, W, Wq refers to expected number in the system, expected number in the queue, expected time in the system, expected time in the queue. The first four columns are results for triangular membership function and the last four are for trapezoidal function. As shown at the table; for both functions, characteristics values decrease up to α=0,65 or α=0,50. Although the values increase outside of this interval, it is shown that the degree of increasement is smaller than the decreasement. It is also shown that for α=1, the values are smaller than the ones where the incresement started. If one wants a possibility of 0,3, than he

decides the possible length of queue is 4,068980 for the triangular membership function and 4,06898015 for the trapezoidal function. Other characteristics mean in the same way.

Table 1. Results for M/F/2 system

α-levels	Lq	L	Wq	W		Lq	L	Wq	W
0,000	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	12,36880
0,050	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	12,36880
0,100	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	12,36880
0,150	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,209943694	12,36880
0,200	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	12,36880
0,250	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	12,36880
0,300	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	12,36880
0,350	4,068980	21,098595	0,299437	12,043684		4,06898015	21,09859484	0,29943694	13,89916
0,400	3,027069	18,484707	0,268972	13,207560		3,02706905	18,48470691	0,26897212	13,89916
0,450	3,027069	18,484707	0,268972	13,207560		3,02706905	18,48470691	0,26897212	13,89916
0,500	3,027069	18,484707	0,268972	13,207560		3,02706905	18,48470691	0,26897212	13,89916
0,550	3,027069	18,484707	0,268972	13,207560		3,02706905	18,48470691	0,26897212	13,70151
0,600	3,130614	18,759684	0,272291	13,068958		3,13061387	18,75968361	0,27229135	13,70151
0,650	3,292777	19,182708	0,277344	12,863931		3,29277714	19,18270827	0,27734365	13,70151
0,700	3,510648	19,737570	0,283873	12,609069		3,51064783	19,73757033	0,28387308	13,70151
0,750	3,214840	18,980522	0,274937	12,960719		3,21484005	18,98052199	0,27493699	13,70151
0,800	3,312463	19,233459	0,277945	12,839973		3,31246307	19,23345931	0,27794543	13,70151
0,850	3,674889	20,146503	0,288616	12,430717		3,67488909	20,14650265	0,28861592	13,70151
0,900	3,354219	19,340693	0,279214	12,789790		335,21936	19,34069307	0,27921392	14,11672
0,950	3,415376	19,496750	0,281053	12,717801		3,4137624	19,49675041	0,28105262	14,11672
1,000	3,362890	19,362890	0,279476	12,779476		3,3688998	19,36288998	0,27947598	

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RESUME

Dans ce travaille, on a pris fuzzy pour les temps de service sur les sistems de la canal de deux queues. Les characteristiques des queues devinerons avec l'aide des fonctions de membreshipes qui sont triangulare et trapesoidaire. A lâ fin les resultats et les solutions sont confrontes.