Estimations of Lorenz Curve and Gini Index in a Pareto Distribution

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1. Introduction

Let $Y_1, Y_2, \ldots, Y_n$ **be a simple random** sample from a Pareto distribution with the pdf:

$$f(y;b) = \frac{b}{b-1} \cdot y^{\frac{2b-1}{1-b}}, \quad 1 < y, \quad 1 < b, \tag{1.1}$$

which has mean b and variance $b(b-1)^2/(2-b)$, if $1 < b < 2$
A Pareto distribution with the pdf (1.1) is a special case of the Pareto distribution with the scale parameter $q = 1$ and the shape parameter $k = b/(b-1)$ (see Johnson(1970)), and the density function is decreasing if $b > 1$. Let $b \equiv \frac{q}{2q-1}$ in a power-function random variable with the pdf $f(x;q) = \frac{q}{1-q} \cdot x^{\frac{2q-1}{1-q}}$, $0 < x < 1$, $1/2 < q < 1$ (see Ali. etal (1999)). Then an inverse of the power-

function random variable has a Pareto distribution with the pdf (1.1). Hung & Bier(1998) considered the properties of the conjugate prior for the non-homogeneous poisson process with a power-function law intensity. Ali, Woo & Yoon(1999) studied the UMVUE for mean and right-tail probability in a power-function distribution , that is, an inverse of a Pareto random variable with the pdf (1.1).

As comparing MSE of MLE and UMVUE of the Lorenz curve and Gini index in a Pareto distribution with pdf (1.1), the UMVUE's of the Lorenz curve and Gini index tend to more efficient than their MLE.

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