

# A Bivariate Dependent Weibull and Competing Risk Model

Cicilia Y. Wada & Mario H. Tarumoto  
*Departament of Statistics*  
*IMECC/UNICAMP*  
*São Paulo, Brazil*  
*cicilia@ime.unicamp.br*

## ABSTRACT

Competing risks analysis with two causes of failure was considered using dependent bivariate model, since independence of times of failures is not a realistic presupposition in many situations. For a single cause of failure the Weibull distribution has been used because its flexibility, allowing increasing, decreasing or constant risk of failure. A bivariate exponential model which is an extension of this univariate model was proposed by Marshall and Olkin (JASA, 1967), with marginals exponentials. This distribution, which is not absolutely continuous, has been studied extensively. For competing risk situation, where the failure times and cause of failure are observed in one individual, if a bivariate model is assumed for the underlying times of failures, a major problem can arise: non-identifiability of the parameters and of the joint survival distribution and its marginals. Also, in competing risk situation, there exist an assumption of the occurrence of the failure for a unique cause, which requires absolutely continuous bivariate distribution for the underlying distribution. Moeschberger (Technometrics, 1974) derived a competing risk model based on the bivariate Weibull distribution of Marshall and Olkin. Thus, we like to investigate for the underlying distribution of the failures times from causes 1 and 2 a bivariate distribution having mainly two properties: flexible specific-hazard functions and having identifiable parameters. Then, a bivariate Weibull distribution was obtained from a modification on the absolutely continuous bivariate Weibull model proposed by Ryu (JASA, 1993), which assure the condition for the estimation of the marginals corresponding to each cause of failure (equality of net and crude hazards). The competing risk model was derived using this modified bivariate Weibull distribution. Identifiability and estimation of its parameters by maximum likelihood method were investigated. Test of hypotheses of its parameters also were studied. Simulation study with samples from this bivariate Weibull distribution, generated using Gibbs and Metropolis-Hasting methods as well as application to real data were performed. Comparisons with the Ryu's bivariate Weibull distribution and with the independent model were conducted.