Bartlett-corrected tests for two-parameter exponential family models

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The asymptotic chi-squared distribution of the likelihood ratio (LR) statistic ω is frequently used to test hypotheses of interest in regression models. However, as the sample size decreases, the use of such a statistic becomes less justifiable. One way of improving the chi-squared approximation to the LR statistic is by using a Bartlett correction. In fact, under mild regularity conditions, the Bartlett correction c guarantees that all moments of the adjusted LR statistic ω^* are equal to those of the asymptotic χ^2 distribution up to order n^{-1}, where n is the sample size. The Bartlett correction c and the modified statistic ω^* are defined by $c = E(\omega)/p$ and $\omega^* = w/c$, where p is the difference of the dimensions of the parameter spaces under the alternative and the null hypothesis and $E(\omega)$ is obtained up to order n^{-1}. The Bartlett corrections are usually effective in bringing the true sizes of the modified statistic ω^* closer to the nominal levels. A method for obtaining c was developed in full generality by Lawley (1956), who showed by a complicated calculation that all cumulants of ω^* agree to order n^{-1} with those of the reference χ^2_p distribution. The disavantage of this method is that it requires certain cumulants of log-likelihood derivatives.

In recent years there has been a renewed interest in Bartlett corrections. Cordeiro (1983, 1987) derived closed-form Bartlett corrections in generalized linear models and discussed improved LR tests. Bartlett corrections for models defined by any one-parameter distribution in which the mean is a known function of a linear combination of unknown parameters were obtained by Cordeiro (1985), who generalized his own results of 1983. Several papers have focused on deriving closed-form Bartlett corrections for specific regression problems. For a detailed account of the applicability of Bartlett corrections, see for example Cribari-Neto and Cordeiro (1996).

The purpose of this paper is to obtain simple Bartlett corrections to improve the LR test of a scalar parameter of two-parameter exponential family models where no cumulants are involved. A simple formula for Bartlett correction for one-parameter exponential models that does not depend on cumulants of log-likelihood derivatives was derived by Cordeiro et al. (1995). Then they applied their result to several distributions in the uniparametric exponential family. The present paper can be therefore viewed as an extension of their paper for two-parameter exponential family models.

Consider a set of n independent and identically distributed random variables $y_1, \ldots, y_n$ with density function

$$\pi(y; \mu, \nu) = \exp\{\alpha_1(\mu, \nu) d_1(y) + \alpha_2(\mu, \nu) d_2(y) - \rho(\mu, \nu) + v(y)\}, \tag{1}$$

where μ and ν are scalar parameters, $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\rho(\cdot, \cdot)$, $d_1(\cdot)$, $d_2(\cdot)$ and $v(\cdot)$ are known functions. If $y_1, \ldots, y_n$ are continuous, π is assumed to be a density with respect to the Lebesgue measure while, if $y_1, \ldots, y_n$ are discrete, π is assumed to be a density with respect to counting measure. We also assume that the support set of (1) is independent of μ and ν and that $\alpha_1(\cdot, \cdot)$, $\alpha_2(\cdot, \cdot)$ and $\rho(\cdot, \cdot)$ have continuous first four derivatives with respect to the parameters μ and ν.

In this work we derive a simple formula for the Bartlett correction to the LR test of a scalar parameter in the exponential model (1) assuming that the parameters μ and ν are globally orthogonal. The formula is simple enough to be used algebrically to obtain closed-form expressions in several special cases since it involves only functions $\alpha_1(\mu, \nu)$, $\alpha_2(\mu, \nu)$ and $\rho(\mu, \nu)$ and their derivatives with respect to the parameters μ and ν. We present a number of distributions of (1) with orthogonal parameters in order to show that our result has a wide range of important applications. We also show that it is always possible to reparametrize the model (1) to achieve the orthogonality between the parameters. We then apply the formula for the Bartlett correction via the reparametrized model to other important tests for which orthogonality does not hold.

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