Optimal Design of Accelerated Degradation Tests under Step-Stress Loading

Sang-Jun Park
Advanced Tech Gr, CS Center, Samsung Electronics Co., LTD
416, Maetan-3Dong, Paldal-Gu, Suwon, Kyungki-Do, 442-742, Korea
juni01@samsung.co.kr

Bong-Jin Yum
Department of Industrial Engineering, Korea Advanced Institute of Science and Technology
373-1, Kusong-Dong, Yusong-Gu, Taejon 305-701, Korea
bjyum@cais.kaist.ac.kr

1. Introduction

Accelerated Tests (ATs) are frequently used to estimate failure time distributions or reliabilities within an affordable amount of time and in an economic way. The stress in an AT can be applied to test units using such methods as constant-stress loading, step-stress loading, progressive-stress loading, etc. Various works exist on designing accelerated life test (ALT) plans under constant-stress or step-stress loading. On the other hand, most of the previous works on designing accelerated degradation test (ADT) plans are concerned with the case where the constant-stress loading method is employed. In this paper, optimal ADT plans under step-stress loading (step-ADTs) are developed under the assumptions of destructive testing and a simple constant-rate relationship between the stress and the performance of a unit.

2. Assumptions

(1) Let U be the performance characteristic of a test unit. Define $Y' = \ln U$. A unit fails when Y' degrades below a specified value g. At any (possibly transformed) stress level s and test duration t, $Y(= \ln U - g)$ follows a normal distribution with mean $m_c(s,t) = a - t\exp(d_1 + d_2 s)$ where a, d_1 and d_2 are unknown constants and standard deviation s does not depend on s and t.

(2) The cumulative exposure model (Nelson, 1990) is assumed for the effect of changing the stress. The test duration and the stress level are standardized such that the test duration becomes 1, and the use and maximum stress level take the values 0 and 1, respectively.

3. Statistically optimal step-ADT plans

We consider a simple step-stress test. That is, only two stress levels are assumed. The test procedure is shown in Figure 1. The problem is to determine s_1, s_2, t_1 and $\{p_i,\ i=0,1,2\}$ such that, $\mathrm{avar}\{\hat{x}_q\}$, asymptotic variance of the MLE of the $q\,th$ quantile of the lifetime distribution at the use the condition, is minimized given n (total number of test units), s_0, s_M and t_M. Under the assumptions in Section 2, $\mathrm{avar}\{\hat{x}_q\}$ can be derived as

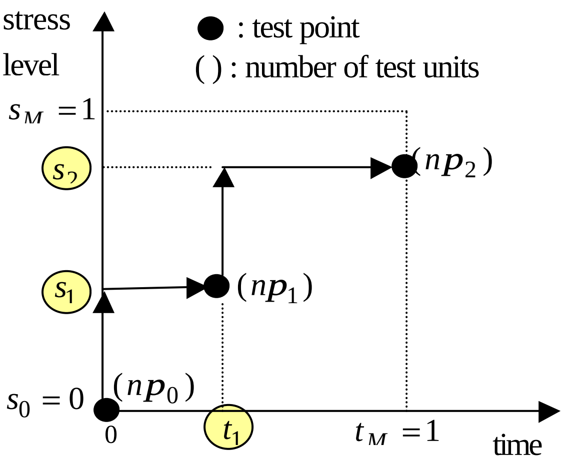

Figure 1. Simple step-stress test

$$\mathrm{avar}\{\hat{x}_q\} = \frac{s^2 Q^2 k^2}{n}\left[\frac{C_0^2}{p_0} + \frac{C_1^2}{p_1} + \frac{C_2^2}{p_2} + \frac{Z_q^2}{2k^2}\right]$$

where $k = Z_q s + a$, $Q = \exp(-d_1)$,

$$C_0 = \left|\frac{1}{k} - \frac{s_2}{(s_2 - s_1)W_1}\right|,$$

$$C_1 = \frac{s_1}{(s_2 - s_1)W_2} + \frac{s_2}{(s_2 - s_1)W_1},\quad C_2 = \frac{s_1}{(s_2 - s_1)W_2},$$

$$W_1 = t_1 \exp(d_1 + d_2 s_1),\quad W_2 = (1 - t_1)\exp(d_1 + d_2 s_2)$$

and Z_q is the $q\,th$ quantile of the standard normal distribution. For any given values of s_1, s_2 and t_1, the optimal values of $p_i\,(i = 0,1,2)$ are determined as $p_i = C_i/(C_0 + C_1 + C_2)$ (Park and Yum, 1997). The other design variables are optimally determined by a grid search method. Let P_{st} be a pre-estimated failure probability at test point $(s,\ t)$. Define $v = \mathrm{avar}\{\hat{x}_q\}$ when $n = 1$. Computational results indicate the following.

(1) $s_2^* = 1$. That is, the optimal high stress level corresponds to the maximum allowable value.

(2) v decreases as P_{00} decreases and/or P_{01} or P_{11} increases.

(3) s_1^* approaches to 0 (i.e., use condition) and t_1^* to 1 (i.e., maximum test time) as P_{00} decreases for given P_{01} and P_{11}, or as P_{11} decreases for given P_{00} and P_{01}, or as P_{01} increases for given P_{00} and P_{11}. Consequently, a two-point test plan, instead of a three-point one, could be optimal in some cases.

The sample size required for a step-ADT can be approximately determined as $n^* \approx vw^2/(x_q \ln c)^2$ where c and f are given constants such that $\Pr\{x_q/c \le \hat{x}_q \le c x_q\} \ge f$, and w is the $(1+f)/2\,th$ quantile of the standard normal distribution.

Finally, an optimal step-ADT plan is compared with the corresponding optimal ADT plan under constant-stress loading in terms of statistical efficiency and the total amount of testing time. Computational results indicate that the proposed step-ADT plan can be used effectively when the amount of testing time is important, but more than one test equipment cannot be employed.

REFERENCES

Nelson, W. (1990). Accelerated Testing: Statistical Models, Test plans, and Data Analysis, Wiley, New York.

PARK, J. I. and YUM, B. J. (1997). Optimal design of accelerated degradation tests for estimating mean lifetime at the use condition, *Engineering Optimization*, 28, pp. 199-230.