A Note on Seasonal Adjustment via Wavelets

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1. Introduction

Seasonal adjustment has been playing an important role in time series analysis of economic variables. In fact, several methods have been proposed to decompose time series data into trend, seasonal, and irregular components. The usual seasonal adjustment procedures range from simple weighted averages to more sophisticated methods based on spectral analysis. The purpose of this paper is to propose a new strategy to seasonally adjust time series data, based on wavelet analysis. The wavelet approach is particularly useful in handling seasonal effects that change gradually over time. An empirical example is also illustrated to implement seasonal adjustment using the simple wavelet methodology.

2. Wavelet Analysis

Wavelet analysis is a comparatively new and powerful mathematical tool for signal processing. In particular, the discrete wavelet transform is useful in decomposing time series data into an orthogonal set of components with different frequencies, and the multiresolution decomposition of wavelet analysis is also useful in handling periodicity in seasonal time series. In this note, we give only a brief overview of these two basic tools of wavelet analysis: DWT and MRD. Readers are referred to Chui (1992) for a thorough review of wavelet analysis and Daubechies (1992) for further technical details. Practical aspects of wavelets are discussed in Bruce and Gao (1996), and an overview on the use of wavelet analysis is given in Lee (1998).

The wavelet representation of any signal or function $f(t)$ in $L^2(\boldsymbol{R})$ can be obtained as a sequence of its projections onto father and mother wavelets given by:

$$(1)\quad f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t).$$

The coefficients $s_{J,k}$ are called the smooth coefficients, representing the underlying smooth behavior of the signal at the coarse scale 2^J. On the other hand, $d_{j,k}$ are called the detailed coefficients, representing deviations from the smooth behavior. Here $d_{J,k}$ describe the coarse scale deviations, and $d_{J-1,k}, \cdots, d_{1,k}$
provide progressively finer scale deviations. Similarly to the wavelet representation (1), a time series can also be expressed in terms of the smooth signal and the detail signals, respectively, as:

$$(2)\quad f(t) = S_J(t) + D_J(t) + D_{J-1}(t) + \cdots + D_1(t)$$

Here the functions $S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t)$ and $D_j(t) = \sum_k d_{j,k}\psi_{J,k}(t)$ represent components of the signal $f(t)$ at different resolutions, and hence the representation (2) is called a multiresolution decomposition.

3. An Illustrative Example

We apply the tools of wavelet analysis discussed above to quarterly data on Korean gross national income. From the summary statistics of the wavelet crystals (not reported here), we can first see that seasonal components represent a large portion of energy, indicating that the income series display strong seasonal variations as well as low-frequency trend. Here the finest scale crystal d1 and corresponding D1 represent seasonal variations occurring within two quarters (or twice a year), and the next finest component d2 and D2 account for variations at a time scale of $2^2 = 4$ quarters (or a year). These high-frequency fluctuations represent seasonal variations in the series, which need to be suppressed to obtain seasonally adjusted data.

Seasonally adjusted data together with unadjusted raw data are displayed in Figure 1. For comparison, also displayed is the officially adjusted data obtained from the ARIMA X-12. The estimates of seasonal components from the two procedures are also compared in Figure 2. Both the seasonally adjusted data and the seasonal components obtained via wavelet decomposition look quite close to those obtained from the usual adjustment procedure except some boundary observations.

—— obtained via wavelets ----- ARIMA X-12

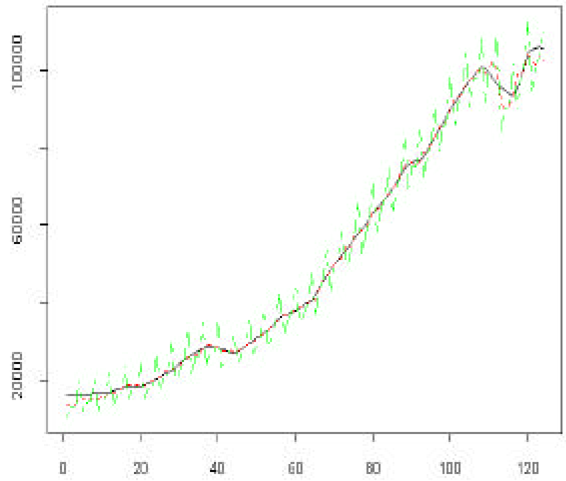

Figure 1. Seasonally adjusted data

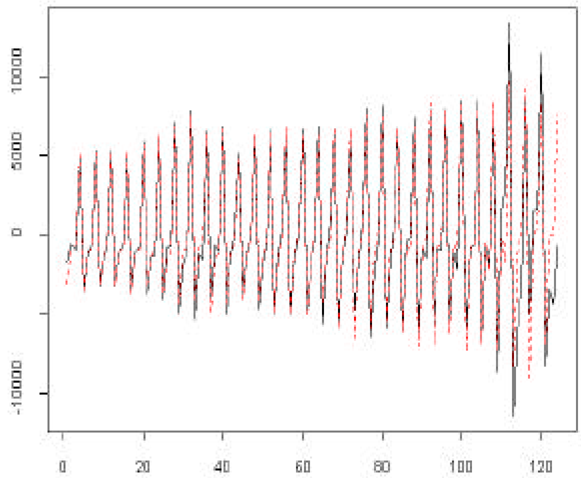

Figure 2. Seasonal components

4. Concluding Remarks

This paper shows that we can obtain seasonally adjusted data by simply applying the wavelet decomposition. However, much work remains to be done. While quarterly data are considered here for ease of exposition, we need to extend this approach to monthly case or other frequency data. As the results are somewhat dependent on boundary conditions of wavelet analysis, further work is required to find optimal boundary treatment rules.

REFERENCE

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RESUME

An attempt is made in this note to implement seasonal adjustment by applying the wavelet decomposition. While much work remains to be done, the simple approach proposed here appears to work quite well. Seasonally adjusted data obtained via the wavelet analysis look quite close to those obtained from the ARIMA X-12 procedure.