A discussion on two ways for measuring growth in nonseasonal time series

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1. The model and proposed estimators

We address the problem of deriving a smooth signal of growth for nonseasonal time series X_t within a model-based approach. We concentrate on annual series for the sake of simplicity, but our results are also valid for seasonal data with purely deterministic seasonality.

We assume that X_t follows an IMA(2,2) model, which is often found appropiate for trending, nonseasonal data:

$$\Delta^2 X_t = \theta(L)\, a_t \qquad a_t \sim \text{Niid}(0, \sigma_a^2) \qquad (1)$$

where $\theta(L)=1-\theta_1 L-\theta_2 L^2$ and invertibility forces θ_1, θ_2 to lie in the triangular region $\theta_2+\theta_1<1$, $\theta_2-\theta_1<1$ and $-1<\theta_2<1$. Direct application of the signal extraction approach leads us to two sensible signals of growth:

1) Under general conditions $X_t=p_{xt}+n_{xt}$; p_{xt} stands for a canonical level trend that is generated by a noninvertible IMA(2,2), let α denote the invertible root of the MA part and σ_b^2 the variance of its innovation; and n_{xt} is white noise. As p_{xt} is unobserved we compute its minimum mean square error (MSE) estimator $\hat{p}_{xt}$ and the first proposed measure is $g_{1t}=\Delta\hat{p}_{xt}$, the growth of the optimal estimator of the level trend, which can be shown to be the output of the linear process (F denotes the forward operator)

$$\Delta\, \theta(F)\, g_{1t} = (\sigma_b^2/\sigma_a^2)\ (1-\alpha L)(1+L)(1-\alpha F)(1+F)\, a_t \qquad (2)$$

2) From (1) the observed growth, $y_t=\Delta x_t$, follows an IMA(1,2) model. If some regularity conditions hold $y_t=p_{yt}+n_{yt}$, with p_{yt} a canonical growth trend generated by a noninvertible IMA(1,1) with innovation variance σ_c^2, and n_{yt} MA(1). The second measure is $g_{2t}=\hat{p}_{yt}$, the minimum MSE estimator of p_{yt}, which is the outcome of the linear process

$$\Delta\, \theta(F)\, g_{2t} = (\sigma_c^2/\sigma_a^2)\ (1+L)(1+F)\, a_t \qquad (3)$$

2. Evaluation criteria

Both candidates seem adequate on theoretical grounds, and to determine which one should be used we put the accent on practical applications. There are two major sources of concern: (a) we are using estimators of the true unobserved signals of interest, Δp_{xt} and p_{yt}, and consequently estimation errors arise; (b) actual inference is based on a finite realization of the series, so some of the estimates are either forecasts or preliminary figures to be revised as new data become available. Therefore we are evaluating the performance of the two measures according to the following criteria: (1) existence of an acceptable decomposition; (2) size of the final estimation error; (3) size of the pure revision errors (we particularize to the concurrent estimator, the one-period revision and the three-period revision); (4) size of the pure forecasting errors (1-period ahead and 3-period ahead); and (5) size of the total error for the same sets of information as in (3) and (4). In (2)-(5) we use the variance to measure the size of the errors.

3. Results

Our analysis indicates that the best measure depends on the characteristics of X_t through the values of (θ_1,θ_2) and the relative importance that the final user assigns to each criterion. Some general rules are: (1) Neither measure is defined for every valid (θ_1,θ_2); and for some (θ_1,θ_2) only g_{1t} is defined. (2) The variance of the final estimation error is lower for g_{1t} unless α is close to 1, which corresponds to the case that X_t is close to a linear trend with a deterministic slope. (3) The revision error of the concurrent estimator mimics what we saw for the final error; there are some changes as new observations arrive, although g_{1t} is still the best choice for most (θ_1,θ_2). When the total error is considered g_{1t} is usually the best election, because the final estimation error tends to dominate the aggregate. (4) g_{2t} is undoubtedly the best choice when the overriding criterion is the pure forecasting error, as it dominates g_{1t} for all combinations of (θ_1,θ_2). Much the same conclusion holds when we consider the total error: g_{2t} is always preferred for 3-period forecasting, while for 1-period ahead it is also the best choice for most valid (θ_1,θ_2).

To sum up, if we leave aside some specific IMA(2,2) models with a positive MA root that tends to cancel out with one difference, the best way for deriving a smooth signal of growth depends on the aims of the final user. For historical analysis and short-term monitoring it seems that the usual practice of extracting a level signal and next computing its growth is adequate. But if the analyst focus on short-term forecasting our results indicate that he/she should better rely on direct extraction of a smooth signal from the observed growth.

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RESUME

Nous comparons les performances de deux méthodes pour obtenir un signal de croissance lisse pour une série chronologique sans variations saisonnières. Nous envisageons le problème dans l'esprit des méthodes basées en modèles et particularisons l'étude au modèle IMA(2,2), qui est approprié pour la plupart des séries avec une décomposition en tendance et bruit. Nos résultats indiquent que pour l'analyse historique ou la surveillance à court terme il va mieux retrouver une tendance de niveau et puis calculer sa croissance; tandis que pour la prévision à court terme on doit utiliser la tendance de croissance estimée directement à partir des données de croissance observées.