Some applications of stochastic approximation to data analysis

Shuhei OGIHARA
Kanazawa University, Faculty of engineering
Kodatsuno 2, Kanazawa shi, 920 Kanazawa, Japan
ogihara@kams.eg.t.kanazawa-u.ac.jp

Shigeyoshi OGAWA
Kanazawa University, Graduate School of Sciences
Kodatsuno 2, Kanazawa shi, 920 Kanazawa, Japan
s_ogawa@t.kanazawa-u.ac.jp

1 Introduction

As is wel-known, the stochastic approximation method introduced by H.Robbins and S.Monro[3] provides a good tool for finding a root of some equation $M(x) = \alpha$ under noisy observation. This method generates random sequence X_n by the following relation

$$X_{n+1} = X_n - a_n(Y(x) - \alpha). \tag{1}$$

Here $Y(x) = M(x) + \text{Noise}$ is a noisy observation data of $M(x)$, and $\{a_n\}$ is a weight sequence. Then it is shown that under suitable assumption of the statistical character of the "noise" term, X_n converges to the unique solution of $M(x) = \alpha$ for each fixed level α in the mean square sense. This method has been studied by many authors, for example J.R.Blum showed convergence of this method with probability one.

2 Application to data analysis

We are interested in the application of this method to the some problems in data analysis such as,

(1) **Change Point Problem** The well known problem in statistics, namely: to detect changing point of distribution function in time series signal. Following B.S.Darkhovski [2], if distribution function changes, the moment of some degree of the random variable sequence changes also. So,

we apply the method to nth moment of $Y_n(X_n)$, then the sequence X_n converges to θ in the mean square.

(2) **Detection of starting point of signal** This problem of detecting a starting point T_0 of some signal $s(t)$ embedded in *i.i.d.* noise, by the series of data.

(3) **Application to parameter estimation** This is a concrete application of Change Point Problem (1). This purpose is to estimate the changing point of the parameter of Brownian emotion which appears in Black-Scholes model.

For these purposes, we need some modification of the Robbins-Monro procedure in order to treat the case of sampled datas at prefixed observation epochs.

References

[1] Blum, J.R.(1954). Approximation methods which converge with probability one. *Ann Math Statist.*, **25**, 382-386.

[2] Robbins,H.,Monro,S (1951). A stochastic approximation method. *Ann. Math. Statist.*, **22**, 400-407.

RESUME

The stochastic approximation method, introduced by H.Robbins and S.Monro in 1951[2] is well known to be a good procedure for finding a root of some equation $M(x) = \alpha$ under noisy observation $Y(x) = M(x)+noise$. We are interested in the application of this method to the problems in data analysis, such as Change Point Problem, and detection of the starting point of signal embeded in noise. In this paper we will discuss a redefined procedure for such problems and show some results of numerical experiments.