On Testing for Separable Correlations of Multivariate Time Series

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1. Introduction

In the analysis of a multivariate stationary time series $\mathbf{Z}_t = (Z_{1t}, \ldots, Z_{rt})'$, it is required to identify its covariance function

$$\mathrm{Cov}(Z_{as}, Z_{bt}) \;\; = \;\; \sigma_a \sigma_b \rho(a, b, t-s), \tag{1}$$

for $a, b = 1, \ldots, r$, where σ_a^2 and σ_b^2 are variances of Z_{at} and Z_{bt} respectively, and $\rho(a, b, t-s)$ is a cross-correlation function of Z_{as} and Z_{bt}. A vector autoregressive and moving average (VARMA) model is one of the most popular models. However identification of (1) for large r is often difficult, since the number of the parameters to be estimated increases formidably as r is larger. Hence it is not feasible to apply them to practical situations if a sample size is small.

Alternatively, separable models have been considered in various fields (e.g. Cressie, 1991; Guyon, 1995) and applied to analyze actual space-time data (e.g. Haslett and Raftery, 1989; Martin, 1990). The separable model is expressed as

$$\mathrm{Cov}(Z_{as}, Z_{bt}) \;\; = \;\; \sigma_a \sigma_b \rho_1(a, b) \rho_2(t-s), \tag{2}$$

where $\rho_1(a, a) = 1$ and $\rho_2(0) = 1$. The separable model means that a cross-correlation function of Z_{as} and Z_{bt} is a product of spatial correlation $\rho_1(a, b)$ and temporal one $\rho_2(t-s)$, if a, b implies locations where observations are obtained. The identification of separable models is easier than that of VARMA models, since we can identify ρ_1 and ρ_2 independently and describe correlation structures of the time series with a fewer number of parameters. Popular candidates for ρ_1 are Matern class (Cressie, 1991) or spatial ARMA models (Guyon, 1995) and those for ρ_2 are univariate ARMA models.

However the separability assumption introduces the same temporal correlation structure in each component series and should be carefully examined in practice (see e.g. the discussion of Haslett and Raftery, 1989). The aim of this report is to provide a testing procedure for the separability assumption so that we can see if the reduction from model (1) to model (2) is justified. Shitan and Brockwell (1995) proposed a method for testing separability by assuming a spatial autoregressive model. However misspecification of the true model can lead to a wrong conclusion.

Hence we propose a nonparametric method based on esitimated spectral density matrices. We construct an estimator for the spectral density matrix under (1) and that under (2) and introduce test statistics which measure the discrepancy between the two estimators.

Then we derive their limiting ditributions under the null hypothesis (2). Next we conduct some computational simulations to see the performance of the test statistics. Finally we apply the proceduer to an empirical data being composed of temperature time series observed at Japanese major 13 observatories.

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RESUME

Notre propos consiste a proposer des statistiques pour examiner la separabilite des series multivariables et a degager leurs caracteristiques asymptotiques. Les statistiques mises a l ’ examen ont ete etablies d ’une maniere non parametrique. C ’est pour cela qu ’elles ont montre la difference entre le controleur qui a examine la matrice de la densite spectrale sans restriction, et celui qui est sous condition de la separabilite. Malgre cette procedure, le resultat est non-parametrique, et l ’examen a un peu de distorsion et la puissance raisonnable.