Estimation for Periodic Autoregressive Models

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Mixed autoregressive (AR) moving-average (MA), ARMA, models with seasonally-varying parameters and orders are known as periodic ARMA (PARMA) models. We consider here the estimation of parameters of periodic AR (PAR) models.

The univariate, w-period, w = 2,3,…, PAR model of seasonally varying orders p(v), v = 1, …, w, denoted by $PAR_w[p(v)]$, can be written for all integers k as

$$X_{kw+v} = \phi^{(v)}_1 X_{kw+v-1} + \ldots + \phi^{(v)}_{p(v)} X_{kw+v-p(v)} + a_{kw+v}$$

where $\{a_{kw+v}\}$ is a white noise process with independently and normally distributed terms with mean zero and periodic variances $\sigma_a^2(v)$. That is, we assume a Gaussian PAR process $\{X_{kw+v}\}$. We further assume that the process is periodic stationary, i.e., the first and second-order moments are finite and periodic in v with period w. The parameters $\phi^{(v)}_1, \ldots, \phi^{(v)}_{p(v)}$ and $\sigma_a^2(v)$ are also periodic. If the means $E(X_{kw+v}) = \mu_v$ are not zero, X_{kw+v} in the equation are to be replaced by $X_{kw+v} - \mu_v$. These means are usually estimated by the sample means. The parametes are to be estimated for a given realization of the process, $\{X_1, \ldots, X_{Nw}\}$, say N years of quarterly data with w = 4.

In this study, exact and conditional maximum likelihood (ML), conditional least-squares (LS), and method-of-moments (MM) estimates are studied. The MM estimates can be obtained from the seasonal Yule-Walker equations. The exact ML estimation is not easy in terms of derivation and solution.

The conditional LS estimates of autoregressive parameters $\phi^{(v)}_1, \ldots, \phi^{(v)}_{p(v)}$ can be obtained in a season-wise manner, and regression methods can be employed directly. The LS method does not provide estimates for error variances $\sigma_a^2(v)$, however, regression type estimates can be used, utilising ML estimates in ordinary regression. It is shown that the conditional ML estimates of the AR parameters are the same as the conditional LS estimates. These are also the same as the MM estimates, except for some initial seasons in which some observations are lost due to end effects. Even in that case, the estimates are not significantly different, being very close for large samples. The conditional ML estimates for variances are the same as the regression-type estimates, and they are also the same as the method of moment estimates, except again for some initial seasons.

Some simulation results are also provided for the various methods of estimation discussed above, with the exception of the exact ML method (Smadi, 1994). The results indicate that the conditional maximum likelihood estimation can be successfully used for all the parameters.

REFERENCES

Smadi, A. A. (1994). Analysis and Development of Statistical Properties of Periodic Autoregressive Moving Average Processes. Ph. D. Dissertation. Department of Statistics, Middle East Technical University, Ankara, Turkey, 126 pp.