Random Ranked Set Samples with Imperfect Judgment Ranking

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SUMMARY

We suggested several unbiased estimators for the population mean using random ranked set samples (RRSS) with errors in ranking estimator. The properties of these estimators are discussed and compare with usual ranked set samples (RSS) with errors in ranking. It turns out that under certain conditions, the newly suggested estimators are more efficient than the usual RSS with errors in ranking.

1. Random Number of Replications with Fixed Number of Classes or Set Size

Let $X_{11}, X_{12}, ..., X_{1n}; X_{21}, X_{22}, ..., X_{2n}; ....; X_{i1}, X_{i2}, ..., X_{in_i}; ...; X_{n1}, X_{n2} ..., X_{nn}$ are independent and identically distributed random variables with cdf F(x), pdf f(x), mean, μ and variance σ^2. Let $v_1, v_2, ...$ be independent random variable taking values from $\Lambda = \{2, 3, ...\}$ and independent of the random variables $X_{ij}, i, j \geq 1$. First we randomly select v_1 sets of size n units, where $v_1 \in \Lambda$. We order the units within each set by judgment order i.e. there is errors in ordering the units within each set. Let $X_{i[1]}, X_{i[2]}, \ldots, X_{i[n]}$, i = 1, 2,.,. v_1 be the judgment order statistics of $X_{i1}, X_{i2}, \ldots, X_{in}$ i = 1, 2, .,. v_1, which are written as $X_{i[j]}, j = 1, 2, ..., n$ to distinguish it from the actual order statistics $X_{i(j)}$. We select for measurement the first judgment order statistics $X^{(1)}_{1[1]}, X^{(1)}_{2[1]}, ..., X^{(1)}_{v_1[1]}$ and let $\overline{y}^{(n)}_{n_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} X^{(1)}_{i[1]}$.

Now we randomly select another v_2 sets of size n units each and order them by judgment order $X_{i[1]}, X_{i[2]}, \ldots, X_{i[n]}$, i = 1, 2, .,. v_2. We select the second judgment order statistics for measurement $X^{(2)}_{1[2]}, X^{(2)}_{2[2]}, \ldots, X^{(2)}_{n_1[2]}$ and denote $\overline{y}^{(n)}_{n_2} = \frac{1}{v_2}\sum_{i=1}^{n_2} X^{(2)}_{i[2]}$.

We repeat this process n times to get $\overline{y}^{(n)}_{n_1}, \overline{y}^{(n)}_{n_2}, \ldots, \overline{y}^{(n)}_{n_n}$. We propose the following as an estimator for the population mean μ

$$\overline{\overline{y}}_n = \frac{1}{n}\sum_{i=1}^{n} \overline{y}^{(n)}_{n_i}$$

The basic properties of the estimator $\overline{\overline{y}}_n$ are:

(a) $\overline{\overline{y}}_n$ is an unbiased estimator of population mean μ with variance

(b) $Var(\overline{\overline{y}}_n) = \frac{1}{n^2}\sum_{i=1}^{n} s^2_{[i]n}\, E(\frac{1}{n_i})$, where $s^2_{[i]n} = Var(X^{(n)}_{[i]j})$.

2. Fixed Number of Replications with Random Set Size or Number of Classes

Let the number of classes or the set size to be a random variable θ taking values from $\Lambda = \{2,3,\ldots\}$ and $k_1, k_2, \ldots, k_q$ are fixed integers. First we select k_1 sets of size θ units, order the units within each set and select from each set the first judgment order statistic for quantification. Let $\bar{y}_{k_1}^{(q)} = \frac{1}{k_1}\sum_{j=1}^{k_1} X_{j[1]}^{(q)}$. Next we select k_2 sets of size θ units, order each set and select from each set the second smallest judgment order statistic for quantification. Then we have $\bar{y}_{k_2}^{(q)} = \frac{1}{k_2}\sum_{j=1}^{k_2} X_{j[2]}^{(q)}$. We repeat the above process θ times to obtain $\bar{y}_{k_1}^{(q)}, \bar{y}_{k_2}^{(q)}, \ldots, \bar{y}_{k_q}^{(q)}$. We propose

$$\overline{\overline{y}}_q = \frac{1}{q}\sum_{i=1}^{q} \bar{y}_{k_i}^{(q)}$$

as an estimator for the population mean μ. The properties $\overline{\overline{y}}_q$ are

(a) $\overline{\overline{y}}_q$ is an unbiased estimator for μ,

(b) with variance $Var(\overline{\overline{y}}_{(q)}) = E[\frac{1}{q^2}\sum_{i=1}^{q}\frac{1}{k_i}s_{[i]q}^2]$.

3. Random number of Replications with Random Set Size or Number of Classes

Let θ, $v_1, v_2, \ldots$ are independent random variables taking values from $\Lambda = \{2,3,\ldots\}$. First we draw randomly v_1 sets of size θ, order the units within each set, and select the first judgment order statistics for quantification from each set. Next we select v_2 sets of size θ, order the units within each set, and select the second judgment order statistics for quantification from each set. We repeat this process θ times. Denote the sample mean of each step by $\bar{y}_{n_i}^{(q)} = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{j[i]}^{(q)}, i = 1, 2, \ldots, q$. Then we propose

$$\overline{\tilde{y}}_q = \frac{1}{q}\sum_{i=1}^{q} \bar{y}_{n_i}^{(q)},$$

as an estimator for m, with variance $Var(\overline{\tilde{y}}_q) = E[\frac{1}{q^2}\sum_{i=1}^{q} s_{[i]q}^2 E(\frac{1}{n_i})]$.

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