An Estimator of Survivor Function Under the Semi-Markov Process with Dependent Censoring

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1. Introduction

Lee and Wolfe (1998) proposed a nonparametric test for the independent censoring assumption and provided a bias-corrected estimator for the survivor function under the proportional hazards model. For this purpose, they proposed a two-stage sampling method in which a small random subset of censored subjects is followed-up until a specified subsequent time. This additional follow-up data for these censored observations allow the assumption of independent censoring to be tested with nonparametric method and a bias of Kaplan-Meier estimator to be adjusted under the proportional hazards model.

Under the dependent censoring model, it is shown by Klein and Moeschbeger(1981) that Kaplan-Meier survivor estimator is biased. Using the extra information about the survival times of censored subjects, a bias-corrected estimator for the survivor function was proposed under the proportional hazards model in Lee and Wolfe (1998).

In this paper, a nonparametric estimator for the survivor function is considered under semi-Markov process in which the hazards rate after censoring depends only on the length from censoring time to the survival time. This procedure is illustrated with an example of a clinical trial analyzed in Lagakos and Williams (1978). Moreover, the simulation results show that the nonparametric estimator tends to have less mean squared error (MSE) under the non-proportional hazards model than Kaplan-Meier and the estimator adjusted under the proportional hazards model.

2. A nonparametric estimator under semi-Markov process

Since the hazard rate depends only on the length of time from censoring to the survival time under the semi-Markov process, the empirical survival probability can be assigned to those who are censored subjects but are not followed up in the second stage. In other words, the probability that censored subjects live longer than the observed time after censoring can be estimated using extra data collected from those who are censored but followed up in the second stage.

At each observed time, a nonparametric estimate for the survivor function can be expressed as the number of subjects who are still alive plus the weight with which the censored subjects would be alive by that time divided by total subjects. The large sample theory for this estimator is derived with its consistency.

3. An Example

In an example of lung cancer trial analyzed by Lagakos and Williams (1978), the nonparametric estimator is compared with the bias-corrected estimator proposed in Lee and Wolfe (1998) and the Kaplan-Meier estimator. The nonparametric estimator shows the similar tendency as the bias-corrected estimator while the Kaplan-Meier estimator behaves far from these two estimators.

4. Simulation study

In order to compare three estimators, the simulation study is performed under the proportional hazards model and under the non-proportional hazards model. For the proportional hazards model, the hazard rate before censoring is assumed to be 1 while the hazard rate after censoring is assumed to be k (>1). On the other hand, the hazard rate under the non-proportional model is assumed to be linear function of time. As is shown in the simulation results, the nonparametric estimate tends to have less mean squared error than the bias-corrected estimate for heavy censoring while the bias-corrected estimate has less bias than the nonparametric estimate in all cases.

Reference

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