Continuous Review Inventory Model Based on Fuzzy Numbers

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1.Introduction

Analysis of an inventory problem is one of the outstanding subjects in operations research. There are various models to describe an inventory process. In crisp inventory models all costs are real numbers and demand is deterministic. In [3], fuzzy set concepts are used for the backorder inventory model by replacing the annual demand, order cost, inventory cost and backorder cost by fuzzy numbers and the economic order quantity is found using function principle. In this paper we will obtain the Continuous Review Inventory Model without Backorder (CRIM), using the fuzzy costs and probabilistic demand. In conclusion, we will show that fuzzy concepts and probability concepts may be used in the same model.

2.Continuous Review Inventory Model without Backorder

In a CRIM the inventory level I, is monitored after every transaction and when I drops to a constant reorder point r, an order is placed. In this model the demand in any given time is a random variable whose probability distribution is stationary. We suppose that units are demanded one at a time or in small quantities. The replenishment lead-time is constant and sufficiently small to compensate if a delay occurs. x denotes the demand with a probability distribution function $f(x)$. All shortages are lost and the shortage cost includes the lost profit. The shortage cost per cycle is $\pi \bar{b}(r)$ where $\bar{b}(r)$ is the expected number of shortages per cycle. The amount of the shortage at the end of the cycle is $b(x,r) = \max[0, x-r]$. The on hand inventory at the end of the cycle is $\bar{a}(r) = r - m + \bar{b}(r)$ where m is the expected demand during a lead-time.

To simplify the model the following variables defined in [1] are used. D is the demand rate in units per year, A is the order cost, C is the unit variable cost of production, h is the inventory carrying cost per unit per cycle, π is the shortage cost per unit short and Q is the order quantity. Therefore the average cost per cycle is

$$A + CQ + h\frac{Q}{D}\left[\frac{Q}{2} + r - m + \bar{b}(r)\right] + p\bar{b}(r) \quad (1)$$

Multiplying (1) by the expected number of cycles per year, the average annual cost is found to be :

$$K(Q,r) = \frac{AD}{Q} + CD + h\left[\frac{Q}{2} + r - m\right] + \left(h + \frac{pD}{Q}\right)\bar{b}(r)$$

The condition $\frac{\partial K}{\partial Q}=0$ leads to $Q=\sqrt{\frac{2D\left[A+\pi\bar{b}(r)\right]}{h}}$

3. Fuzzy CRIM

In real life situations, the costs defined in CRIM may not be constants. To reflect this vagueness we should consider all the costs in fuzzy sense. So we replace all costs by fuzzy numbers. Suppose $\tilde{A}=(a_1,a_2,a_3,a_4)$ is trapezoid fuzzy order cost, is $\tilde{C}=(c_1,c_2,c_3,c_4)$ trapezoid fuzzy unit variable cost of production, $\tilde{\Pi}=(\pi_1,\pi_2,\pi_3,\pi_4)$ is trapezoid fuzzy shortage cost per unit short and $\tilde{H}=(h_1,h_2,h_3,h_4)$ is trapezoid fuzzy inventory carrying cost per unit per cycle where a_i, c_i, π_i, h_i (i = 1, 2, 3, 4) are real variables and satisfy the conditions $0\le a_1\le a_2\le a_3\le a_4$, $0\le c_1\le c_2\le c_3\le c_4$, $0\le \pi_1\le \pi_2\le \pi_3\le \pi_4$ and $0\le h_1\le h_2\le h_3\le h_4$. By using the function principle [3] we obtain the trapezoid fuzzy average annual cost is $\tilde{K}=(k_1,k_2,k_3,k_4)$ where

$$k_i=a_i\frac{D}{Q}+c_iD+h_i\left[\frac{Q}{2}+r-m\right]+\left(h_i+\frac{\pi_iD}{Q}\right)\bar{b}(r)\qquad i=1,2,3,4$$

Under the condition $0\le k_1\le k_2\le k_m\le k_3\le k_4$ we found the economic order quantity with defuzzification. We obtained the median of trapezoid fuzzy average annual cost as shown below:

$$k_m=\frac{1}{4}\left\{\sum_{i=1}^{4}\left(a_i\frac{D}{Q}+c_iD+h_i\left[\frac{Q}{2}+r-m\right]+\left(h_i+\frac{\pi_iD}{Q}\right)\bar{b}(r)\right)\right\}$$

The condition $\frac{\partial K}{\partial Q}=0$ leads to $Q=\sqrt{\frac{2D\left(\sum_{i=1}^{4}a_i+\sum_{i=1}^{4}\pi_i\bar{b}(r)\right)}{\sum_{i=1}^{4}h_i}}$

4.Conclusions

In this article we used fuzzy sense in crisp CRIM without backorder, fuzzified all the costs in this model and determine the demand as a random variable. As a result, we show that both fuzzy concept and probability concept may be used in the same model to describe the real situations in an inventory problem.

REFERENCES

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RESUME

Dans ce travaille, on approche avec la base de "fuzzy" ombre au modele Continu Passe en Revue Inventaire. On emploie le prensibe fonction de Chen, on réuni l' approche fuzzy etý' approche probabilité.