A Bayesian method of estimating mortality rates in small areas

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1. Introduction

The Ministry of Health and Welfare (MHW) introduced a Bayesian method of estimating mortality rates for small areas in Japan more than ten years ago. Until then, since mortality rates are not stable in small areas, only the figures for larger areas were published.

Let P_i be the standard population for the i th age group and Q_i be the death rate of standard population. Further, for the i th age group in the target small area, let p_i and d_i denote the population and the observed number of death. Then using the observed mortality rate $q_i = d_i/p_i$, the directly age-adjusted mortality rate (DAR) and the standardized mortality ratio (SMR) are obtained by

$$\mathrm{DAR} = \frac{\sum P_i q_i}{\sum P_i} \qquad \mathrm{SMR} = \frac{\sum d_i}{\sum p_i Q_i}$$

Although SMR is usually more stable than DAR, both of them suffer from the variation of actual death rate in small areas. To cope with this difficulty, the MHW introduced the following simple method, which they termed a Bayesian method.

Suppose the number of death in a particular sex-age group in the target area is distributed like binomial $d_i \sim B(p_i, \theta_i)$ and is independently of the death in other groups. Then MHW introduced a prior distribution for the parameter θ, that is, the true mortality rate. As the prior distribution they assume beta distribution $\theta_i \sim beta(\alpha_i, \beta_i)$, and the hyper parameters are determined by matching the mean and variance to those obtained from the figures for the larger area containing the small areas. It is a simple procedure to derive the posterior distributions of θ_i for each of small areas. Finally, instead of observed death d_i, they use the posterior expectation $p_i E(\theta_i \mid d_i)$.

As is easily imagined, the method achieved stable estimation for small areas than directly using SMR or DAR.

2. An extended method

Although the MHW method is simple, we can point out its limited usefulness. The mortality rates are similar for small areas within a larger area, but the similarity is stronger if the two small areas are closer. We need to incorporate the relative strength of the similarity. It is also well known that mortality rates vary among age groups, but the changes are relatively small for adjacent age groups. The MHW method does not provided a solution to deal with such realistic information in estimation.

The author proposes a method that is derived from Yoshizoe (1987) where the binary regression model of any functional form can be estimated by assuming smoothness for the response function. Here, the only modification is to cope with the binomial, rather than the binary response variable that was explicitly dealt with in Yoshizoe (1987).

The model is as follows. Let θ indicate the true mortality rate of a certain area-age group. Then we assume the most general form by

$$\theta(x) = F(\beta(x)^T x) \tag{1}$$

where F is the logistic distribution function $F(x) = 1/(1+e^{-x})$, x represents a three dimensional variable that consists of the latitude and longitude of the center of each small area, and the age group indicator (0-4, 5-9 and so on up to 100-).

In order to express relative closeness of θ, we introduce the prior distribution that satisfies the following relation, which you can call "stochastic Lipschitz condition."

$$\Pr\{\|\beta(x) - \beta(x')\| < K\|x - x'\|\} = 1 - \varepsilon \quad (\forall x, x') \tag{2}$$

where $\|x - x'\| = d(x, x')$ is a quasi distance. The following conditions are simplified and practical version.

(A) var[β(x) − β(x')] is proportional to $d(x, x')^2$

(B) var[β(x)] = $\xi^{-1}\Omega \quad (\xi \to 0)$

(C) $\beta(x)$ is distributed as normal

Based on this prior distribution, we can evaluate the mortality rates in small areas more realistically than the MHW method.

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