Robust Statistical Analysis of Financial Models for the Short Term Rate

Rosario Dell'Aquila
University of Southern Switzerland, Institute of Finance
Via Buffi 13
Lugano, Switzerland
Rosario.Dell'aquila@lu.unisi.ch

Elvezio Ronchetti
University of Geneva, Department of Econometrics
Boulevard Carl Vogt 102,
Geneva, Switzerland
Elvezio.Ronchetti@metri.unige.ch

Fabio Trojani
University of Southern Switzerland, Institute of Finance
Via Buffi 13
Lugano, Switzerland
Fabio.Trojani@lu.unisi.ch

1. Introduction

We re-examine using the robust Generalized Method of Moments (RGMM, Ronchetti and Trojani (2001)) the findings of Chan et al. (1992) (hereafter CKLS), who tested several single factor models for the short interest rate on US treasury bills data. Using RGMM all models are clearly rejected. The different results compared to those of the classical GMM are explained in a simple sensitivity experiment, where the classical findings are found to be highly unstable. Looking at the influential points found by the RGMM we note a cluster of observations in the 1979-1982 period, that is well-known to coincide with a temporary change in the monetary policy of the Federal Reserve. Based on these robust weights we conclude that no CKLS model is supported by the data for the whole CKLS sample period. Section 2 introduces basic definitions, Section 3 presents empirical results, while Section 4 concludes.

2. RGMM Analysis of Models for the Short Rate Process

CKLS consider a class of continuous-time stochastic processes for the short term interest rate, whose discrete version is

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + \epsilon_t \quad , \quad \epsilon_t^2 = \sigma^2 r_{t-1}^{2\gamma} + \eta_t \quad , \tag{1}$$

where ϵ_t, η_t are some corresponding error terms. α and β characterize the linear drift component, σ is the instantaneous volatility parameter, while γ measures the sensitivity of volatility with respect to the current interest rate level r_t. Constraints on $(\alpha, \beta, \sigma, \gamma)$ can be imposed giving a well-known class of one-factor short rate models (cf. CKLS and Table 1 below).

Table 1. Short rate models and parametric constraints

ME	VA	CIR	DO	GBM	BS	VR	CEV
$\beta = 0, \gamma = 0$	$\gamma = 0$	$\gamma = 0.5$	$\alpha = 0, \beta = 0, \gamma = 1$	$\alpha = 0, \gamma = 1$	$\gamma = 1$	$\gamma = 1.5$	$\alpha = 0$

The CKLS orthogonality conditions suited for a GMM estimation of (1) are

$$E(\epsilon_t) = 0 \qquad E(\epsilon_t r_{t-1}) = 0 \qquad E(\eta_t) = 0 \qquad E(\eta_t r_{t-1}) = 0 \quad . \tag{2}$$

For a set of weighting matrices $(W_n)_{n \in \mathbb{N}}$ converging to a positive definite matrix W_0, a GMM estimator (GMME) $\tilde{\theta} := (\tilde{\theta}_n)_{n \in \mathbb{N}}$ of a parameter θ_0 is a sequence of solutions to the optimization problem (cf. Hansen (1982)) $\min_{\theta \in \Theta} E_{F_n} h'(X_1; \theta) W_n E_{F_n} h(X_1; \theta)$,where $F_n := \frac{1}{n} \sum_{i=1}^{n} \delta_{X_i}$ is the empirical distribution of the observations $X_1, ..., X_n$, and δ_x denotes the point mass distribution at $x \in \mathbb{R}^N$. The set of orthogonality conditions (2) is defined by an unbounded orthogonality function h and implies non-robust GMM estimators and tests (see Ronchetti and Trojani (2001) for details). The general idea behind the construction of a RGMM estimator is therefore to construct a weighted version of h that is bounded and that can be again interpreted as a set of GMM orthogonality conditions. For a given constant $c > \sqrt{H}$ define $h_c^{A,\tau}(x, \theta) = A[h(x; \theta) - \tau] \cdot w_c(A[h(x; \theta) - \tau])$, where $w_c(y) := \min(1, \frac{c}{\|y\|})$ for $y \neq 0$, and $w_c(0) := 1$. The nonsingular matrix A and the vector τ are determined by the implicit equations:

$$E_F h_c^{A,\tau}(X_1, \theta_0) = 0 \quad , \quad E_{F_n} h_c^{A,\tau}(X_1, \theta_0) h_c'^{A,\tau}(X_1, \theta_0) = I \quad . \tag{3}$$

The RGMME $\widetilde{\theta}_c^{A,\tau}$ is obtained by iteratively computing the GMME associated to the bounded orthogonality function $h_c^{A,\tau}$. Note that the first expected value in (3) is with respect to the given reference model distribution and not with respect to the empirical distribution of the data. In this paper we use as a reference model for (1) one with conditionally normally distributed errors ε_t. Finally, notice that the above RGMME induces tests with stable level and power over contaminated neighboorhods of the given reference model (cf. again Ronchetti and Trojani (2001) for details).

3. Data and Empirical Results

We use the CKLS data set, that is the 1-month Treasury Bill series taken from the 12-month Fama Treasury Bill Files included in the CRSP monthly Government Bonds Files. These are 307 monthly observations from June 1964 to December 1989. In Table 2 the p-values of the classical and the robust GMM statistics are presented[1].

[1] To preserve space we focus on the results obtained for Hansen's (1982) statistics.

Table 2. GMM and RGMM p-values of Hansen's test

	ME	VA	CIR	DO	GBM	BS	VR	CEV
GMM	(0.034)	(0.009)	(0.027)	(0.133)	(0.206)	(0.137)	(0.098)	(0.084)
RGMM	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.145)	(0.08)

Based on the robust Hansen's statistics all models with $\gamma \leq 1$ (ME to BS in the table) are rejected at a 5% significance level. This is at odds with the classical results, where models with $\gamma = 1$ (DO, GBM, BS) could not be rejected. Moreover, the significance of rejection of the single models is high, and the p-value of Hansen's test changes when comparing the classical with the robust GMM tests. For the GBM model, for example, it goes from about 0.2 to about 0.00003. On the other hand models with $\gamma > 1$ are not rejected by standard significance levels[2].

Fourteen influential observations were identified automatically through the robust weights. 10 of them are in the 1979-1982 period which is well-known to coincide with a temporary monetary regime change in the policy of the Federal Reserve. Consistently with the results of others studies (cf. for instance Bliss and Smith (1999)) we interpret this as a hint of a model misspecification of the CKLS models, caused by the change in the monetary policy of the Federal Reserve. Further inspection shows patterns of influential observations that are similar to regimes of high volatility found for example in Gray (1996). These influential points correspond to the first oil crisis and the October 1987 stock market crash.

To understand the robustness problem of the classical GMM specification test as well as the difference between a cross-validation technique for the treatment of single outliers (as used for instance in Bliss and Smith (1999)) and the RGMM, we perform some sensitivity analyses of the $p-$value of Hansen's statistic under some simple forms of model contaminations. To preserve space we present a straightforward sensitivity analysis where we move the 6 most influential observations identified (a contamination of about ϵ=2%) over intervals of about +/- 40 basispoints around the 6 observed non contaminated values of the short rates in the CKLS data set. The results are presented in Table 3 where the maximal and the minimal p–values obtained are listed for each model.

Table 3. Maximal and minimal p-value of Hansen's test under contamination of the 6 most influential observations

	ME	VA	CIR	DO	GBM	BS	VR	CEV
Min	0.0082	0.0016	0.0059	0.0726	0.1211	0.0497	0.0392	0.0645
Max	0.1017	0.0325	0.0734	0.1811	0.2593	0.2338	0.1688	0.1072

The maximal sensitivity of the p-values of Hansen's statistics is very high, causing in some cases

[2] However, a detailed inspection of the influential points identified by RGMM suggests that also these models and the unrestricted CKLS one are misspecified. Furthermore, the results for $\gamma > 1$ are given only for completeness and for comparison with CKLS, since under this parameter choice the standard GMM asymptotic inference does not hold (cf. Broze, Scaillet and Zakoian (1995)).

(ME, CIR, BS, VR) a reversion of the test decision. For instance, in the Brennan Schwartz model the p-values go from 0.0497 to 0.2338. This shows that when testing contaminated models by GMM even small changes of some observations can have a strong impact on the test results.

4. Conclusions

In this paper we focused on an application of RGMM within the standard CKLS framework. We found unstable GMM statistics causing unreliable classical GMM procedures and model selections. On the other side, RGMM model selection procedures perform well and offer a valid complement to the classical strategy. They make general misspecification structures in the CKLS models transparent by identifying a cluster of influential points before 1982 and some isolated outliers after this date. Based on this evidence no CKLS model is able to fit the interest rate series under scrutiny for the whole sample period.

REFERENCES

Bliss, R.R., and D.C. Smith (1999). The Elasticity of Interest Rate Volatility: Chan, Karolyi, Longstaff, and Sanders revisited, *Journal of Risk*, **1**, 21-46.

Broze, L., Scaillet, O., and Zakoian, J. (1995). Testing for Continuous-Time Models of the Short-Term Interest Rate, *Journal of Empirical Finance*, **2**, 199-223.

Chan, K. G., G. A. Karloyi, F. A. Longstaff and A. B. Sanders (1992). An Empirical Comparison of Alternative Models of the Short-Term Rate, *Journal of Finance*, **47**, 1209-1227.

Gray, S. (1996). Modelling the Conditional Distribution of Interest Rates as a Regime–Switching Model, *Journal of Financial Economics*, **42**, 27-62.

Hansen, L.P. (1982). Large Sample Properties of Generalized Method of Moments Estimators, *Econometrica*, **50**, 1029-1054.

Ronchetti, E., and F. Trojani (2001). Robust Inference with GMM Estimators, *Journal of Econometrics*, **101**, 36-69.

RESUME

Dans ce papier, nous testons avec la Méthode Robuste des Moments Généralisés (RGMM, Ronchetti and Trojani (2001)) les modèles monofactoriels de taux d'interêt dans Chan. et. al (1992). Dans cette application nous observons une haute instabilité de la Méthode classique des Moments Généralisés, ce qui donne des estimations et des tests classiques peux fiables. Au contraire, la méthode robuste donne des bons resultats en faisant ressortire une mauvaise spécification des modèles et en identifiant une concentration d'observations influentes avant 1982. Cette évidence mentre que tout modèle CKLS est mal specifié dans l'application considerée.