A new redescending M-estimating function

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1. Proposed M-Estimation Function

New redescending function is based on minimization of L_2 distance of a model density and it's density estimator. Let g_θ be a family of probability densities indexed by θ. The minimum L_2 distance estimator $\hat{\theta}$ is defined by a statistical quantity minimizing L_2 distance, which is a solution to

$$\nabla_\theta \mu(p, g_\theta) = \nabla_\theta \int (p(x) - g_\theta(x))^2 dx = 0, \qquad (1)$$

where we assume $p(x), g_\theta(x) \in L_2$ and ∇_θ represents a derivative with respect to θ. Since we have $\int g_\theta(x) \nabla_\theta g_\theta(x) dx = (1/2) \nabla_\theta \int g_\theta^2(x) dx = 0$, if θ is a location parameter, the equation (1) becomes

$$\int p(x) \nabla_\theta g_\theta(x) dx = \nabla_\theta \int p(x) g_\theta(x) dx = 0. \qquad (2)$$

Suppose $p(x)$ is a kernel is a Gaussian and a model $g_\theta(x)$ is the normal with mean μ and variance σ^2, then $\int (1/h) K\{(x - X_i)/h\} g_\mu(x) dx$, a convolution of a Gaussian kernel and a normal density, is the normal with mean μ and variance $h^2 + \sigma^2$. After dropping unnecessary constants, the equation (2) becomes

$$\psi(X_i; T_n) = (X_i - \mu) \exp[-(X_i - \mu)^2 / 2(h^2 + \sigma^2)] \Big|_{\mu = T_n}.$$

That is, we have

$$\psi_r(t) = t \exp[t/2r^2] \text{ for } t \in (-\infty, \infty), \text{where } r^2 = h^2 + \sigma^2. \qquad (3)$$

2. Conclusions

The proposed function is basically a redescending type function, and we could show that estimator by this new function attain the same level of robustness as the existing redescending M-estimators, but have less asymptotic variance than others. We have focused on estimating a location parameter in this study, but the method can be extended for a scale estimation.

REFERENCES

Andrews DF, Bickel PJ, Hampel FR, Huber PJ, Rogers WH, & Tukey JW (1972) Robust Estimates of Location: Survey and Advances. Princeton Univ. Press, Princeton NJ

Beaton AE & Tukey JW (1974) Outlier in Statistical Data. Wiley, New York

Silverman BW (1986) Density Estimation for Statistics and Data Analysis. Chapman & Hall, London

Hampel FR, Ronchetti EM, Rousseeuw PJ, & Stahel WA (1986) Robust Statistics, the Approach Based on Influence Functions. John Wiley & Sons, New York

RESUME

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Ph.D. (Statistics), Univ. of Texas-Austin ,U.S.A.,1993;