A new redescending $M$-estimating function

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1. Proposed $M$-Estimation Function

New redescending function is based on minimization of $L_2$ distance of a model density and it’s density estimator. Let $g_\theta$ be a family of probability densities indexed by $\theta$. The minimum $L_2$ distance estimator $\hat{\theta}$ is defined by a statistical quantity minimizing $L_2$ distance, which is a solution to

$$\nabla_\theta \mu(p, g_\theta) = \nabla_\theta \int (p(x) - g_\theta(x))^2 dx = 0,$$  \hspace{1cm} (1)

where we assume $p(x), g_\theta(x) \in L_2$ and $\nabla_\theta$ represents a derivative with respect to $\theta$. Since we have $\int g_\theta(x) \nabla_\theta g_\theta(x) dx = (1/2) \nabla_\theta \int g_\theta^2(x) dx = 0$, if $\theta$ is a location parameter, the equation (1) becomes

$$\int p(x) \nabla_\theta g_\theta(x) dx = \nabla_\theta \int p(x) g_\theta(x) dx = 0.$$  \hspace{1cm} (2)

Suppose $p(x)$ is a kernel is a Gaussian and a model $g_\theta(x)$ is the normal with mean $\mu$ and variance $\sigma^2$, then $\int (1/h)K\{(x - X_i)/h\}g_\mu(x)dx$, a convolution of a Gaussian kernel and a normal density, is the normal with mean $\mu$ and variance $h^2 + \sigma^2$. After dropping unnecessary constants, the equation (2) becomes

$$\psi(X_i; T_n) = (X_i - \mu) \exp[-(X_i - \mu)^2/2(h^2 + \sigma^2)]|_{\mu=T_n}.$$  

That is, we have

$$\psi_r(t) = t \exp[t/2r^2] \text{ for } t \in (-\infty, \infty), \text{ where } r^2 = h^2 + \sigma^2.$$  \hspace{1cm} (3)
2. Conclusions

The proposed function is basically a redescending type function, and we could show that estimator by this new function attain the same level of robustness as the existing redescending M-estimators, but have less asymptotic variance than others. We have focused on estimating a location parameter in this study, but the method can be extended for a scale estimation.

REFERENCES


RESUME

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