Power Comparisons of the Discontinuous Trend Unit Root Tests

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1. **Unit root tests allowing for multiple breaks**

Suppose that the time series observations $1,2,\cdots,T$ can be broken into K-intervals to allow for the (K-1) breaks in the deterministic trend. Let the number of observations at the end of K intervals is T_i, $i=1,\ \cdots,\ K$, and it is assumed that each break is given as the first observation after the end of an interval. Step dummy variables are denoted by DU_{it} which is 1 for $t=T_{i-1}+1,\ \cdots,T_i$, and zero otherwise, for $i=1,\ \cdots,K$, and $T_0=0$. The sample size is T which is also equal to T_K under the assumption of (K-1) breaks in the deterministic trend, and $\sum_{1,K} DU_{it}=1$ for all t. Shock dummy variables are denoted D_{it} which is one when $t=T_{i-1}+1$ and zero otherwise, $i=2,\ \cdots,K$.

The null hypothesis is $H_0:\phi=0$. Let the regression equation of the test be

(1) $$y_t=m(t)+u_t,\quad u_t=(1+\phi)u_{t-1}+\varepsilon_t,\ u_1=0$$

where $m(t)=\sum_{i=1}^{K}(\alpha_i^*+\beta_i^*t)DU_{it}$. The initial value is $y_1=m(1)=\alpha_1^*+\beta_1^*$.

Following Dickey and Fuller (1981), we rewrite (1) as

(2) $$\{1-(1+\phi)L\}\ u_t=\{1-(1+\phi)L\}\ \{y_t-m(t)\}=\varepsilon_t.\quad t=2,\cdots,T$$

under the alternative hypothesis. The equation (2) is expanded as

(3) $$\Delta y_t=\phi\ y_{t-1}+\sum_{i=1}^{K}\{\beta_i^*+\phi(\beta_i^*-\alpha_i^*)\}DU_{it}-\sum_{i=1}^{K}\beta_i^*\phi tDU_{it}$$
$$+\sum_{i=2}^{K}(1+\phi)\{\alpha_i^*-\alpha_{i-1}^*+(\beta_i^*-\beta_{i-1}^*)T_{i-1}\}D_{it}+\varepsilon_t,$$

and the error term is $ID(0,\sigma^2)$. This equation is redefined as

(4) $$\Delta y_t\equiv\phi\ y_{t-1}+\sum_{i=1}^{K}\alpha_iDU_{it}+\sum_{i=1}^{K}\beta_i\ tDU_{it}+\sum_{i=2}^{K}\gamma_iD_{it}+\varepsilon_t,\qquad t=2,\cdots,T$$

where the coefficients in (4) bear nonlinear restrictions as can be seen by comparing (3) and (4). The equation (4) is an auxiliary regression from which the *t* ratio on ϕ coefficient is

(6) $$\hat{\tau}_{CM} = \frac{\sum_{i=1}^{K}\sum_{t=1}^{T} y_{t-1}^{+}\,\Delta y_t (DU_{it} - D_{it})}{\sqrt{\hat{\sigma}^2 \sum_{i=1}^{K} \sum_{t=1}^{T} {y_{t-1}^{+}}^2 (DU_{it} - D_{it})}}$$

where y_{t-1}^{+} is the residual of regressing y_{t-1} on all other explanatory variables in (4).

2. Nonlinear regression test

Here we give nonlinear regression tests for $H_0{:}\phi = 0$. Firstly, we estimate coefficients $\theta^* = (\phi, \beta_1^*, \cdots, \beta_K^*, \alpha_1^*, \cdots, \alpha_K^*)$ in the nonlinear equation (3) by a nonlinear least squares method, and calculate the sum of squared residuals denoted RSS_A. We also estimate the null coefficients $\theta_0^* = (0, \beta_1^*, \cdots, \beta_K^*, \alpha_1^*, \cdots, \alpha_K^*)$ in (3), and calculate RSS_0.

(10) $$F_{CM} = \frac{1}{\hat{\sigma}^2}(RSS_0 - RSS_A)$$

is used as a test statistic where $\hat{\sigma}^2 = RSS_A / (T - 2K - 1)$. Under the null hypothesis, F ratio converges to the square of the limit of (6).

3. Perron Test

ϕ is consistently estimated by the OLS estimator of ϕ in the regression equation

(12) $$\Delta \hat{u}_t = \phi\, \hat{u}_{t-1} + \sum_{i=2}^{K} \gamma_i D_{it} + \text{error},$$

where $\hat{u}_t$ is the OLS residual calculated from $y_t = m(t) + u_t$. The shock dummy variables such as D_{it} are used to omit observations at the break points in estimation. The *t* ratio is

(13) $$\hat{\tau}_{CM}^{\ *} = \frac{\sum_{t=1}^{T} \hat{u}_{t-1} \Delta \hat{u}_t (DU_{it} - D_{it})}{\hat{\sigma}\sqrt{\sum_{i=1}^{T} {\hat{u}_{t-1}}^2 (DU_{it} - D_{it})}}$$

which converges to τ_{CM} under the null hypothesis $H_0 : \phi = 0$. It may be of interest to find that this test also requires the shock dummy variables.

4. Power Comparison and Conclusion

The two of the four unit root test statistics are listed as follows: $\hat{\tau}_{CM}$ of (6) for testing $H_0 : \phi = 0$ in (1); $\hat{\tau}_{CM}^{\ *}$ of (13) for testing $H_0 : \phi = 0$ in (1).

Small-sample powers are compared by the simulation technique. Through simulations, small-sample power of the Dickey-Fuller type *t* test is found to be greater than the Perron test. Similar results hold with the coefficient test.