Power Comparisons of the Discontinuous Trend Unit Root Tests

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1. Unit root tests allowing for multiple breaks

Suppose that the time series observations 1, 2, \ldots, T can be broken into K-intervals to allow for the (K-1) breaks in the deterministic trend. Let the number of observations at the end of K intervals is $T_i$, $i = 1, \ldots, K$, and it is assumed that each break is given as the first observation after the end of an interval. Step dummy variables are denoted by $DU_{it}$ which is 1 for $t = T_{i-1} + 1, \ldots, T_i$, and zero otherwise, for $i = 1, \ldots, K$, and $T_0 = 0$. The sample size is $T$ which is also equal to $T_K$ under the assumption of (K-1) breaks in the deterministic trend, and $\sum_{i=1}^{K} DU_{it} = 1$ for all $t$. Shock dummy variables are denoted $D_{it}$ which is one when $t = T_{i-1} + 1$ and zero otherwise, $i = 2, \ldots, K$.

The null hypothesis is $H_0: \phi = 0$. Let the regression equation of the test be

\begin{equation}
  y_t = m(t) + u_t, \quad u_t = (1 + \phi)u_{t-1} + \varepsilon_t, \quad u_1 = 0
\end{equation}

where $m(t) = \sum_{i=1}^{K} (\alpha_i^* + \beta_i^* t)DU_{it}$. The initial value is $y_1 = m(1) = \alpha_1^* + \beta_1^*$.

Following Dickey and Fuller (1981), we rewrite (1) as

\begin{equation}
  \{1 - (1 + \phi)L\} u_t = \{1 - (1 + \phi)L\} \{y_t - m(t)\} = \varepsilon_t, \quad t = 2, \ldots, T
\end{equation}

under the alternative hypothesis. The equation (2) is expanded as

\begin{equation}
  \Delta y_t = \phi y_{t-1} + \sum_{i=1}^{K} \{\beta_i^* + \phi(\beta_i^* - \alpha_i^*)\}DU_{it} - \sum_{i=1}^{K} \beta_i^* tDU_{it} + \sum_{i=2}^{K} (1 + \phi)(\alpha_i^* - \alpha_{i-1}^* + (\beta_i^* - \beta_{i-1}^*)T_{i-1})D_{it} + \varepsilon_t,
\end{equation}

and the error term is ID(0, $\sigma^2$). This equation is redefined as

\begin{equation}
  \Delta y_t = \phi y_{t-1} + \sum_{i=1}^{K} \alpha_i DU_{it} + \sum_{i=1}^{K} \beta_i tDU_{it} + \sum_{i=2}^{K} \gamma_i D_{it} + \varepsilon_t, \quad t = 2, \ldots, T
\end{equation}

where the coefficients in (4) bear nonlinear restrictions as can be seen by comparing (3) and (4). The equation (4) is an auxiliary regression from which the $t$ ratio on $\phi$ coefficient is...
where $y_{t-1}^*$ is the residual of regressing $y_{t-1}$ on all other explanatory variables in (4).

2. **Nonlinear regression test**

Here we give nonlinear regression tests for $H_0 : \phi = 0$. Firstly, we estimate coefficients $\theta^* = (\phi^*, \beta_1^*, \ldots, \beta_K^*, \alpha_1^*, \ldots, \alpha_K^*)$ in the nonlinear equation (3) by a nonlinear least squares method, and calculate the sum of squared residuals denoted $RSS_\Lambda$. We also estimate the null coefficients $\theta_0^* = (0, \beta_1^*, \ldots, \beta_K^*, \alpha_1^*, \ldots, \alpha_K^*)$ in (3), and calculate $RSS_0$.

$$F_{CM} = \frac{1}{\sigma^2} (RSS_0 - RSS_\Lambda)$$

is used as a test statistic where $\sigma^2 = RSS_\Lambda / (T - 2K - 1)$. Under the null hypothesis, $F$ ratio converges to the square of the limit of (6).

3. **Perron Test**

$\phi$ is consistently estimated by the OLS estimator of $\phi$ in the regression equation

$$\Delta \hat{u}_t = \phi \hat{u}_{t-1} + \sum_{i=2}^{K} \gamma_i D_{it} + \text{error},$$

where $\hat{u}_t$ is the OLS residual calculated from $y_t = m(t) + u_t$. The shock dummy variables such as $D_{it}$ are used to omit observations at the break points in estimation. The $t$ ratio is

$$\hat{\tau}_{CM}^* = \frac{\sum_{t=1}^{T} \hat{u}_{t-1} \Delta \hat{u}_t (DU_{it} - D_{it})}{\hat{\sigma} \sqrt{\sum_{t=1}^{T} \hat{u}_{t-1}^2 (DU_{it} - D_{it})}}$$

which converges to $\tau_{CM}$ under the null hypothesis $H_0 : \phi = 0$. It may be of interest to find that this test also requires the shock dummy variables.

4. **Power Comparison and Conclusion**

The two of the four unit root test statistics are listed as follows: $\hat{\tau}_{CM}$ of (6) for testing $H_0 : \phi = 0$ in (1); $\hat{\tau}_{CM}^*$ of (13) for testing $H_0 : \phi = 0$ in (1).

Small-sample powers are compared by the simulation technique. Through simulations, small-sample power of the Dickey-Fuller type $t$ test is found to be greater than the Perron test. Similar results hold with the coefficient test.