Optimal Design of Accelerated Life Tests with Two Stresses and Periodic Inspections

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1. Introduction

Accelerated life tests (ALTs) are widely used to quickly obtain information on the lifetimes of highly reliable products. In some ALTs, two stresses (e.g., temperature and voltage) are employed to further reduce the amount of testing time (Park and Yum, 1996), and, in addition, inspections are conducted periodically for administrative convenience. In this article, optimal two-stress ALT plans are developed under the assumptions of exponentially distributed lifetimes of test items, Type-I censoring, and periodic inspections at an accelerated condition. The optimality criterion adopted is to minimize the asymptotic variance of the maximum likelihood estimator (MLE) of the log mean lifetime at the use condition, and the design variables include the low level of each stress and the proportion of test items allocated to each stress condition. Continuous and periodic inspection schemes are compared in terms of statistical efficiency, and the computational results indicate that the latter can be used without serious loss in statistical efficiency.

2. Model

We assume the following models.

(1) Two stresses (T and V) are employed.

(2) At any T and V, lifetimes (Y) of test items are independent and follow an exponential distribution, of which the cumulative distribution function is given by

$$F(y) = 1 - \exp(-y/\alpha) \quad \text{for} \quad y > 0 \text{ and } \alpha > 0.$$

(3) The mean lifetime α depends on T and V according to the following relationship.

$$\alpha(T,V) = \exp(\beta_0 + \beta_1 T + \beta_2 V + \beta_3 TV).$$

For simplicity and without loss of generality, T and V are regarded as standardized variables such that the use level is 0 and the high level is 1. We also standardize the censoring time to be 1, and all the original time-related variables are divided by the censoring time.

3. Optimal ALT Plans

Optimal design of an ALT plan includes determining the number and location of test points and the proportion of test items allocated to each test point such that the asymptotic variance of the MLE of the log mean lifetime at the use condition is minimized. In this article, we consider four test points as candidates as shown in Fig. 1. This factorial arrangement enables us to test possible interaction between T and V. Then, a two-step optimization procedure is adopted to determine the location of test points (i.e., t and v in Fig. 1) and the proportion of test items allocated to each test point, π_{ij} ($i = t$, 1 and $j = v$, 1). In the first step, for given t and v, optimal π_{ij} is determined as

$$\pi_{ij}^{*} = \sqrt{\tau_{ij}} \Big/ \left(\sqrt{\tau_{tv}} + \sqrt{\tau_{t1}} + \sqrt{\tau_{1v}} + \sqrt{\tau_{11}}\right),$$

where $\tau_{ij} = c_{ij}^2 \dfrac{\left\{\exp\left(\Delta/\alpha_{ij}\right) - 1\right\}^2}{\left(\Delta/\alpha_{ij}\right)^2 \exp\left(\Delta/\alpha_{ij}\right)\left(1 - P_{ij,M+1}\right)}$, $c_{tv} = 1/\delta$,

$c_{t1} = -v/\delta$, $c_{1v} = -t/\delta$, $c_{11} = tv/\delta$, $\delta = (1-t)(1-v)$, $P_{ij,M+1} = \exp\left(-1/\alpha_{ij}\right)$ for $i = t$, 1 and $j = v$, 1, M is the total number of inspections at each test point, and Δ is the interval between two adjacent inspections. In the second step, optimal t and v are determined by the Powell conjugate direction search (Press, *et al.*, 1988). Let $P_{ij} (i = 0,1 \; and \; j = 0,1)$ be a pre-estimated failure probability at test point (i, j). Then, optimal ALT plans for various combinations of $P_{ij} (i = 0,1 \; and \; j = 0,1)$ values behave as follows.

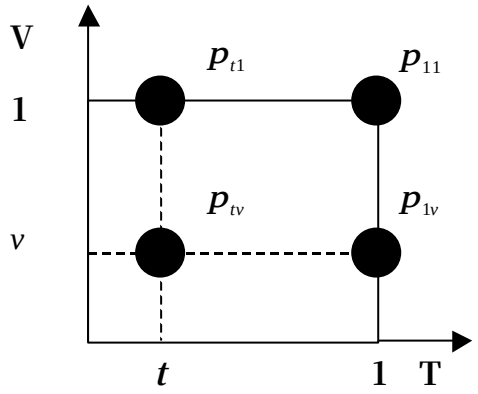

Figure 1. Location of test points

(1) Proportions of test items allocated to the four test points are such that $\pi_{tv}^{*} > \pi_{1v}^{*}, \pi_{t1}^{*} > \pi_{11}^{*}$. In addition, if $P_{10} = P_{01}$, then $t^{*} = v^{*}$ and $\pi_{t1}^{*} = \pi_{1v}^{*}$, and if $P_{10} > P_{01}$, then $t^{*} > v^{*}$ and $\pi_{t1}^{*} < \pi_{1v}^{*}$.

(2) Two extreme cases were observed. In the first case, the test is conducted at the use condition only. This occurs when the acceleration effects of both stresses are small. In the second case, only one stress is employed and the other becomes ineffective. This occurs when the acceleration effect of a stress is substantially higher than that of the other.

Additionally, the present optimal ALT plans are compared with those under continuous inspection (Park and Yum, 1996) in terms of statistical efficiency, and it was found that periodic inspection can be used without serious loss of statistical efficiency.

REFERENCES

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