# To Design Experiments in Blocks of Small size

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## 1.Introduction

Blocking technique has been utilized for setting up experiments to improve accuracy of the inference on experimental results. Under the consideration of blocking, usually experimental runs are obtained based on the defining relations or the assignment of factors in an orthogonal array, and then partitioned equally into blocks of a given size. The block size can be varied depending on experimental situations. Sun, Wu and Chen(1997) discussed optimal blocking schemes for two-level factorial experiments in blocks of all the possible sizes. Rosenbaum (1999) explained the possible use of blocks of various size when blocking compound dispersion experiments. In this article we consider fractional factorial experiments in blocks of small size. When one arranges experimental runs into blocks, his major concern is the confounding of any effects he plans to investigate with blocking effects. The confounding causes inaccurate estimates of the corresponding effects.

Based on the experience from analysis of experimental data, some high-order interactions might not be as important as main-effect or low-order interactions. We just consider two-way interactions. The accuracy of estimates depends on the blocking method used for experimental runs. In this article, we propose methods to deal with the accuracy problem and the size problem. In order to propose methods for obtaining more accurate estimates for main-effect and two-way interactions, we review a method of constructing orthogonal arrays for two-level experiments and the assignment on an orthogonal array for establishing experimental runs in the next section. In section three, we suggest a blocking scheme and three assignments rules to obtain experimental runs in blocks of different sizes for complete factorial experiments.

## 2. Orthogonal Arrays and Assignment Rules

Let $OA_N(2^{N-1})$ be an orthogonal array each of whose N-1 orthogonal columns contains equal numbers of l's and –l's, where $N=2^k$ for some integer k. Define $AB=(a_ib_i)$ to be the product of vector $A=(a_i)$ and $B=(b_i)$ and denote k Nx1 vectors $X_1=(-1, -1,\dots, -1, 1, 1,\dots, 1)^T$, $X_2=(-1,\dots, -1, 1,\dots, 1, -1,\dots, -1, 1,\dots, 1)^T$,…, and $X_k=(-1, 1, -1, 1, -1, \dots, -1, 1)^T$. Use the multiplication of these columns, we can obtain an $OA_N(2^{N-1})$ orthogonal array. For our purpose, we use an orthogonal array to design a complete n-factorial experiment. When n factors are assigned to an appropriate set of columns in the array, we obtain N treatment combinations in an order, the row order. When $\{X_1, X_2,\dots, X_{n-1}\}$ are used for blocking these combinations into N/2 blocks, the first block contains the first two runs, the second contains the next two runs and so on. Based on the construction of the orthogonal array, all the columns containing column $X_n$ are orthogonal to the blocking columns. This suggests that all the effects given by those columns are free of blocking effects.

Let $X=X_1X_2\dots X_n$ , assign n factors $A_1,\dots, A_n$ to columns,

$$X,\ XX_2, XX_3, \dots, XX_n \qquad (1)$$

When blocking columns $\{X_2, X_3, \ldots, X_n\}$ called $G_1$ blocking, are utilized , all the effects given by those columns containing $X_1$ are free of blocking. The assignment (1) using $G_1$ blocking results in a design. Obviously the resulting experiments are in blocks of size two and is better than those given in Draper and Guttman(1997). In fact, the effects from odd-way interactions are free of blocking effects and the even-way interactions are confounded with blocking effects.

For the experiments in block of size four, we need another assignment. Assign factors $A_1,\ldots, A_n$ to

$$X, XX_1, XX_2, XX_3, XX_4X_1, XX_5X_2, XX_6,\ldots, XX_{n-1}X_q \ , \qquad (2)$$

where q=(n-1)mod(3). When $\{X_3, X_4,\ldots, X_n\}$ called $G_2$ blocking is used, all the main effects are estimable and free of blocking effects. With this design we optimize the number of two-way interaction free of blocking. For the experiments in block of size eight, we use $G_3$ blocking $\{X_4, X_5, \ldots, X_n\}$ for the assignment of factors $A_1,\ldots, A_n$ to

$$X, XX_1, XX_2, XX_3, XX_4X_1X_2, XX_5X_1X_3, XX_6X_2X_3, XX_7, \ldots, XX_{n-1}W_q \ , \qquad (3)$$

where $W_q$= $X_0$ ,$X_1$,$X_2$, $X_1X_2$, $X_1X_3$, or $X_2X_3$ depending on (n-1)mod(7) equal to 0,1 …, or 6 . Surely all the main effects in this design are estimable and free of blocking effects. There are only few two-way interaction confounded with blocking effects.

## REFERENCE


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## RESUME


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