Bivariate Gamma Type Quasi-Likelihood with Constant Co rrelation Structure

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1. Introduction

Since the idea of two-variate gamma type distribution has been proposed by Kibble (19 41), many authors discuss similar types of theories of multivariate gamma distribution includ ing bivariate case [see Krishnaiah(1977, 1985), Krishnaiah and Rao(1961), Krishnamoorthy a nd Parthasarathy(1951), Rosen(1977)]. Recent research reviews the progressmade to date on this issue (Kotz et al., 2000). A multivariate gamma distribution would beneeded to get a multivariate distribution model with greater flexibility in dependence struc- ture and indices of dispersion through an m-variate compound distribution. However there isno known multiv ariate gamma distribution with convenient form for the distribution that leads to a simple form for compound model(Joe, 1997). We would propose new gamma type bivariate distrib ution model through the extension of Wedderburn's theory of quasi-likelihood in order to co nquer this difficulty.

2. Condition of Quasi-likelihood Equation

An unbiased estimating function $g(q, y)$ is defined to be a function of the data y and p-dimensional parameter q having zero mean for all q, that is, $E_q\{g(q,Y)\}=0$ for all q. A quasiscore function, $q(q, y)$ as defined by Wedderburn(1974), is a linear unbiased estimating function based only on the first two moments of observation. One of the purposes of an estimating function is to obtain an estimate q of the parameter from data y as a root of equation.
The efficient score function is by definition the gradient of the log-likelihood which is regarded as a potential function, but the quasiscore function often is not a gradient of any potential function, that is, fails to have a symmetric derivative matrix of ¶q/¶q even at q. Such a score function cannot be the gradient of any potential function on the parameter space, that is, there is no "quasi-likelihood" $Q(q, y)$ such that ¶Q/¶q = $q(q, y)$ [see McCullagh and Nelder (1989) and McCullagh (1991)].

3. Extension to Bivariate Quasi-likelihood with Gamma Type Margins

According to the definition of conservative quasiscore (Li et al., 1994), there should be a potential function Q_g such that ¶Q_g /¶q_s = g_s . This is equivalent to the statement that the line integral of g over Θ is path independent. If g is continuously differentiable, an equivalent version is $\partial\gamma^r/\partial\theta_s = \partial\gamma^s/\partial\theta_r$ for r, s = 1, .., p, which we call integrability condition. McCullagh and Nelder (1989) point out that the condition is not always satisfied in multi-variate case. We will show how to handle the integrable factor to find the quasi-likelihood to satisfy the integrability condition, and propose a gamma type quasi-likelihood for bivariate case.

REFERENCE

[1] Joe, H. (1997). *Multivariate Models and Dependence Concepts,* Chapman and Hall.

[2] Kibble, W. F. (1941). A two-variate gamma type distribution, Sankhya, 5, 137-150.

[3] Kotz, S., Balakrishnan, N. and Johnson, N. L. (2000). Multivariate Gamma Distributions, In *Continuous Multivariate Distributions*, 2nd ed., vol. 1: Models and Applications. 431-483. John Wiley & Sons.

[4] Krishnaiah, P. R. (1977). On generalized multivariate gamma type distributions and their applications in reliability, In *The Theory and Applications of Reliability,* Edited by Tsokos, C. P. and Shimi, I. N., eds. 475-494. Academic Press.

[5] Krishnaiah, P. R. (1985). Encyclopedia of Statistical Sciences, vol. 6, pp.63-66. John Wiley & Sons.

[6] Krishnaiah, P. R. and Rao, M. M. (1961). Remark on a multivariate gamma distribution, American Mathematical Monthly, 68, 342-346.

[7] Krishnamoorthy, A. S. and Parthasarathy, M. (1951). A multivariate gamma-type distribution, Ann. Math.Statist., 22, 549-557. Correction (1960). 31, 229.

[8] Li, B. and McCullagh, P. (1994). Potential Functions and conservative estimating functions, The Annals of Statistics, 22, 340-356.

[9] McCullagh, P. (1991). Quasi-likelihood and estimating functions. In *Statistical Theory and Modelling*: *In Honour of Sir David Cox*(D.V.Hinkley, N.Reid and E.J. Snell, eds.) 265-286. Chapman and Hall.

[10]McCullagh, P and Nelder, J. A. (1989). *Generalized Linear Models*, 2nd ed. Chapman and Hall.

[11]Rosen, T. (1977). Multivariate Gamma Distributions (Update), Encyclopedia of Statistical Sciences, Update Volume 1, pp. 419-425. John Wiley & Sons.

[12]Wedderburn, R. W. M. (1974). Quasi-likelihood functions, generalized linear models, and the Gauss-Newton method. Biometrika 61, 439-447.